International Journal of Business and Management Sciences **E ISSN**: 2708 – 4337 **P ISSN:** 2708 – 4329
Available online at http://www.ijbmsarchive.com
International Journal of Business and Management Sciences
Volume 06 (01), 2025
Received, 06 January, 2025, *Accepted, 09 March, 2025,*
Online, 10 March, 2025

The Inflation-Finance Feedback Loop: A Theoretical Investigation of policy Intermediation's Role in Pakistan's Inflation Dynamics

[1]Dr. Bashir Ahmad, [2]Dr. Altaf Hussain, [3] Dr. Ikram Ullah, [4]Fozia Khan

Keywords:
Money and Interest Rates, Policy Interaction, Financial Intermediaries, Tax-Exemption, Inflation determination.

ABSTRACT

The financial intermediaries play a pivotal role in the transmission mechanism of both fiscal and monetary policy for determining the rate of inflation. Moreover, in developing economies like Pakistan, fiscal dominance is a norm and tax exemption for certain sectors is a pattern. This study explicitly model to gauge the response of inflation for such situations, incorporating the acknowledged role of intermediaries in the frame of policy coordination by taking into account the risk taking behavior of deposit holders and loan advancing strategies of banking industry.

INTRODUCTION

The main course of actions for achieving the macro-economic objectives of stable inflation are Fiscal and Monetary policies. Both the policies have exclusive spheres of action, however, each policy orb affect the other policy's actions, endorsing the need of efficient policy coordination for attaining steady inflation. A non-coordinating behavior on part of any policy will leave both the policies ineffective in achieving these goals. Disproportionate fiscal deficits, intended to bump up the aggregate demand can stimulate inflationary pressure in an economy, resulting in nominal depreciation of the local currency and worsening balance of payment situation-5.

A situation where the fiscal policy actions are more active in comparison to monetary policy, leads to macro-economic instability and inefficient monetary policy outcomes. This situation leads to financial flux, high exchange rate fluctuation and bleak economic circumstances. In plain words, missing policy coordination leads to poor economic performance and can potentially destabilize public expectations, distracting the monetary policy efforts of price stability. In the words of Quoting Worrell (2000), less independence of central bank in

[1]*Assistant Professor, Department of Economics, ICP. Email: b.ahmad3@icp.edu.pk*
[2] *Assistant Professor, Department of Economics, ICP. Email: altaf@icp.edu.pk*
[3] *Assistant Professor, Department of Economic, University of Malakand. Email: ikramullah@uom.edu.pk (Corresponding Author)*
[4] *PhD Scholar, Department of Economics, ICP. Email: foxeekhan@gmail.com*

[5] Unlike developed economies, monetary policy in developing economies is typically dominated by fiscal considerations. Central banks have often been required to finance public sector deficits. Such subordination of monetary policy to fiscal considerations leads to an inflationary bias. For more details, see Worrell (2000).

developing economies directs to more fiscal policy activism in comparison to monetary policy. In such situation, the central banks are required to finance public sector deficits at high costs which lead to high inflationary outcomes.

Research studies conducted by Barro (1990) and Jones and Manuelli (1995) assigns a more important and noted role to the government fiscal policies in inflation dynamics of an economy. The literature about finding the relationship between inflation, public policies and budget deficit is affluent enough to describe the core problem and suggest alternative solutions. However, the subject literature fails to link together the fiscal policies to inflation and output taking into account the role of financial intermediaries. The inclusion of banking industry in such analysis is vitally important because of two land mark reasons:

First, all financial transactions and investments in an economy are made through financial intermediaries. Secondly, banking industry in a country is the sole field of game for the monetary authorities. Thus, inclusion of banking sector to gauge the collective efforts of monetary policy and fiscal policies for reducing soaring inflation and bumping up the plunging is of crucial importance. It will not only put light on the complex nature of investment decisions in boom and bust, but will also help analyze the depositors and investors response to changes in fiscal and monetary policies.

We follow different models on the subject for developing model for this study. The concept of fickle depositors in our model is taken from Diamond and Dybvig (1983) model, where the authors developed a framework to assess the policy issues associated with the economies of banking industry. A single bank in their study is taken as representative agent of banking industry, which is examined in the perspectives of banking contracts that can avert the stern economic smash up started due to pre-mature withdrawal of deposits.

While developing our model for endogenous, the model of Bencivenga and Smith (1993) is consulted. This model is focuses the role of financial intermediaries in addressing the liquidity requirements of agents and its ability to play role by shifting saving boxes to capital accumulation. Our study also takes helps from the model of Azariadis & Smith (1996), where the money growth and output relationship is studied by using the asymmetrically modified money growth. Their model focuses on determination of equilibrium in the economy and envisages the pattern of long term growth when money supply is lastingly swelling and the initial level of inflation is low.

The model of Hung (2003) is also considered as it studies the role of general price level in the context of financial development when inflation affects growth of the economy. The model specifies that informational imperfection is the footing of financial market and loan transactions are done on the basis of money. The study concludes that in the economies where the initial level of inflation is too high, the relative large public spending can adversely affect financial development and increase the general price level besides dipping down output growth. Following Schreft & Smith (1997), monetary aspects where liquidity in the system is introduced by financial intermediaries are incorporated in our model. Their model not only defines the role of banks in channelizing savings of households but also explains the role of government as instigator of the bonds. For our model, the more interesting part is its squeezing of private resources by government and the consequent crowding out. The multiple steady state equilibriums which can emerge from a constant increase of money stock are analyzed in the model. Production side of the model is designed on the basis of Romer (1986) model where the production process of goods is externally stanched.

The inflation based revenue generation for financing budget expenditure is more a monetary phenomenon while the direct taxation of household's income is a fiscal phenomenon. This rationale advocates the solidification of the fact that the two policies can't be settled in separate spheres. Thus, taking into consideration both the monetary and fiscal policy set up together, the frame of model is developed with the endeavor to find out the responses of inflation to fiscal shocks, when the investment and saving decisions are concocted by financial intermediaries.

To intricate, this study tries to observe the interaction of the two economic policies (fiscal and monetary) on the face of household's risk taking behavior for their deposits in bank. As, economic literature provides enough evidence that high inflation prospects not only depend on tax and expenditure level but also on dispersion of bank's portfolio to different kind of projects. The conclusion of the study opens new horizons of discussion as some of the results are not in line with the conventional views on the subject, including odd relationship of taxes and output. Inflation responds positively to seigniorages, while debt financing reacts negatively to every positive change in seigniorages. Growth toes the same upward positive line if expenditure is increased by government on account of bulky budget deficit. While defining the relationship between inflation and growth, a negative outcome is witnessed.

Motivation of the study

Rising nations like Pakistan can struggle with slow development, inflationary pressures, and fiscal supremacy. Policy attempts notwithstanding indicate the need of a more thorough theoretical investigation of policy interactions as the ongoing inability to attain macroeconomic stability points to Determining inflation results mostly depends on financial intermediaries, who also act as channels of policy distribution. Still, a lot of the research ignores the mathematical modeling of banking dynamics within systems of fiscal and monetary policy. By building a theoretical model combining depositor risk behavior, financial intermediaries' portfolio decisions, and the intricate coordination of fiscal and monetary authorities, this work aims to close that void.

Research Gap

Although policy interactions have a lot of research underlined, few studies model the function of financial intermediaries using a simply quantitative and theoretical methodology. In Pakistan, where fiscal dominance sometimes compromises monetary policy, knowledge of how actions in the banking industry balance policy impacts is vital. The literature suffers a major void when actual research on the risk behavior of the depositor and its effect on investment allocation is lacking. This work explicitly includes financial intermediary dynamics and develops a thorough theoretical model to replicate policy outcomes, hence bridging that gap.

Theoretical and Conceptual Frame

Following Romer's (1986) endogenous growth theory, the study builds an OLG model including banking dynamics founded on Diamond & Dybvig (1983). The model records government budgetary restrictions, bank lending policies, and risk preferences of depositors. It analyzes inflation reactions to fiscal and monetary policy shocks, then analytically investigates several equilibrium trajectories. Through the simulation of policy interactions, the model reveals situations whereby policy coordination improves stability, so providing important theoretical insights for developing countries.

Objectives of the Study

- Using overlapping generations (OLG) and endogenous growth frameworks, the study build a mathematical model that reflects the interaction between fiscal and monetary policy through financial intermediaries.
- By means of mathematical testing of several policy scenarios, identification of prerequisites inflation stability.
- To investigate how risk preferences of depositors affect the portfolio allocation decisions of banks and thus macroeconomic results.
- To offer theoretical analysis of the structural difficulties facing emerging nations—especially Pakistan—as well as how well coordinated policies could lower inflation.

LITERATURE REVIEW

Policy interaction's theoretical underpinnings have been much investigated. The 'credit channel' idea was first proposed by Bernanke and Gertler (1995) and demonstrated via balance-sheet impacts how monetary policy influences credit availability. Barro (1990) and Jones & Manuelli

(1995) established how government expenditure shapes long-term growth and inflation. While Bencivenga and Smith (1993) investigated how financial intermediaries channel savings into effective capital accumulation, Diamond and Dybvig (1983) offered a mathematical framework for understanding depositor liquidity preferences.

More lately, research has enlarged these ideas. Emphasizing the importance of policy alignment, Bolhuis, Koosakul, and Shenai (2024) devised the "fiscal R-star" model to find debt-stabilizing interest rates. Sheard (2023) underlined how financial middlemen help to cushion against economic shocks so enabling ideal capital allocation. Investigating leverage cycles and liquidity restrictions, Adrian and Shin (2020) and Brunnermeier and Sannikov (2021) demonstrated how dynamics in the banking sector impact policy efficacy. Agha & Khan (2006) analyzed the inflationary consequences of fiscal deficits in Pakistan in the framework of developing economies; Hung (2003) showed that too much public expenditure may impede financial development and growth by means of inflationary pressures.

Keeping in view this literature, we developed an OLG model for a representative developing economy in general and economy of Pakistan in particular. We define the role of currency transaction in an atmosphere of Structural segregation and elusive communication. The role of banks is defined in the context of households' desire for keeping money as cash in hands or illiquid assets in response of fiscal policy fabrication. Moreover, the role of government in the model is designed in such a way where the order of its expenditure is determined exogenously and the revenues segment of fiscal catalogue is determined on the basis of bonds, direct taxation and seigniorages.

THE MODEL

Output, Firms, Capital Returns and Wage Rate

i. Output

At the production front, the economy is considered single good and firms operate in perfectly competitive environment. The output of the representative firm is equal to Y_t, while the total output depends not only on 'private' capital allocated to it, but also on the infrastructure support the public ventures provide. The amount of firm's output has a higher if the success of projects initiated (both public and private) is high.

$$(1) \qquad Y_t = f\ (g_k\, p_k\, h_l) = \text{Ƥ}\left(\bar{g}_k^{1-\Psi_{\sigma c}}\, p_{kt}^{\Psi_{\sigma c}}\, h_{lt}^{1-\Psi_{\sigma c}}\right)$$

$\bar{g}$ is public capital, p_{Kt} is Private Capital in time period t and h_{lt} is Labor in time period t.

The model specifies no leisure-work trade off dimensions as each worker toils for fix hours. Productivity parameter Ƥ is assumed to be greater than zero, and a certain portion of $\bar{g}_k$ depends on private savings allocated to public sector projects by the financial intermediaries as investment.. It should be noted that $\bar{g}$ corresponds to public investment for production industry related infrastructure. Besides, it provides positive spillover externalities and provides the atmosphere suitable for flourishing the production industry in the country.

The total earnings are distributed in two mutually exclusive poles of consumption and saving. Each unit saved is equal to one unit of consumption given up for investment and consequently earns one unit of capital in the next time period.

ii. Firms

There is perfect competition, firms are price takers and tries to maximize profit. Total cost of the firms includes the wages remunerated and rent paid. The firms' revenue depends on the amount of output it produces and sells in one time period. Firm's operates on the basis of zero profit (total revenues are equal to its cost). Labor supply in the economy is fully utilized and firms' marginal costs are equal to marginal revenues as there is perfectly competitive environment.

Total earning in the economy depends on the wages of labor and rent the capital stock pay back. And Equation 1 will turn out to be as follows:

(2) $$Y_t = f\ (g_k\, p_k\, h_l) = \text{Ƥ}\left(p_{Kt}^{\Psi_{\sigma c}}\, h_{lt}^{1-\Psi_{\sigma c}}\right)$$

It can be made more specific by taking the derivation of firm's profit with respect to 'capital' (p_{Kt}) and 'labour' $\left(h_{lt}^{\Psi_{\sigma c}}\right)$.

(3) $$\text{Ƥ}\, F(\bar{g}_k\, p_{Kt}\, h_{lt}) = \mathrm{R}_w\left(h_{lt}^{\Psi_{\sigma c}}\right) + \mathrm{k}_r(p_{Kt})$$

iii. Capital Returns and Wage Rate

Wage rate R_w and Capital return K_r are given below in equation 4 and 5 by taking the derivatives of equation (1) with respect to labor and capital, respectively we get

$$\mathrm{R}_w = \{1 - \Psi_{\sigma c}\}\,(\text{Ƥ})\frac{p_{kt}^{\Psi_{\sigma c}} / h_{lt}^{\Psi_{\sigma c}}}{\bar{g}_k^{\Psi_{\sigma c}-1}}$$

Labor supply is inelastic, and labor force is fully utilized $(h_{lt} = 1)$.

As public investment is infrastructure related and part of total investment, therefore, total capital is equal to both private and public capital assortment[6]. Further, the model deem public capital[7] $(\bar{g}_k)$ equal to the private investment p_{kt}. Based on these assumptions, we can further simplify wage rate as follows:

---

[6] $\bar{g}_{kt} + p_{kt} = p_{Kt}$

[7] a spread out externality in the production function

$$\{1-\Psi_{\sigma c}\}\,(\text{Ƥ})\frac{{p_{kt}}^{\Psi_{\sigma c+1-\Psi_{\sigma c}}}}{h_{lt}^{\Psi_{\sigma c}}}$$

(4) $$\mathrm{R}_w = \{1-\Psi_{\sigma c}\}\,(\text{Ƥ})p_{KT}$$

R_w = Real wage Rate

In the same fashion we can find the capital rent as follows:

$$\mathrm{k}_r = \Psi_{\sigma c}\,\text{Ƥ}\frac{\bar{g}_k^{1-\Psi_{\sigma c}}}{z_{kl}^{\Psi_{\sigma c}}}$$

$$= \Psi_{\sigma c}\,\text{Ƥ}\frac{p_{kt}^{1-\Psi_{\sigma c}}}{z_{kl}^{\Psi_{\sigma c}}}$$

(5) $$\mathrm{k}_r = \text{Ƥ}(\Psi_{\sigma c}) = \mathrm{E}_{\mathrm{k}}^{\mathrm{gT}}$$

K_r is Real return on capital, while $z_{kl}^{\Psi_{\sigma c}}$ is Capital labor ratio $(p_{Kt}^{\Psi_{\sigma c}}/h_{lt}^{\Psi_{\sigma c}})$

Total Savings as Deposits, Returns on Total Saving, Bank's Allocations and Depositor's Utilities

i. Total Savings and Deposits (Bank portfolio) Dispersal

To shape the model more in line with the contemporary banking structure, the factual aspects have been given a more reflective connotation by defining different types of returns. Although, the division and pattern of banking ventures in Pakistani economy are quite unique in certain aspects and saving options are more complex. Pakistani society is divided in two broader categories when it comes to categorization of population on truly financial bases. Unskilled or low income groups, whose main saving brands are largely defined to be currency and demand deposits; and well-heeled class or skilled group, who are either not interested in equity market due to religious reasons or doesn't want to coup up with such an equity market where the desired sophistication is missing.

However, for minimalism reasons, total aggregate public savings are divided in two major categories: savings invested for the purchase of government bonds and money kept as deposits in banks. Both modes are of crucial importance for accurately determining the interest earnings (returns on savings) the banks offered to households.

Bonds are government issued certificates bearing (mostly) a fixed interest rate and issued by government for raising money to finance budget deficit. While, banks invest total deposits in capital market, money market and a fraction S_r^r is reserved in cash to meet the CRR obligations. Money market investments are implicitly specified to be time unbound demand deposits, while the capital investment is meant to be time bound deposits.

Banks also invest a certain fraction 'b^{g}' of the total deposits for the government's bonds purchase

$$S_t^{GB} = b^g(S_t^d).$$

Under such circumstances, the total loan-able bank's portfolio B_p^l is equal to

$$B_p^l = S_t^d\,(1 - S_t^{GB} - S_r^r)$$

Where, B_p^l is the loan-able portfolio that the bank can lend after S_r^r (cash reserve requirement) and S_t^{GB}(credit used for the purchase of bonds) are deduced from the aggregate total savings (S_t^d).

We take up total loan-able bank's portfolios (B_p^l) to be initially equal to aggregate savings S_t^d, as there is no pre-specified allocation of bank portfolio for bonds purchase and the reserve requirement S_r^r is negligible.

Bank divides its total deposits S_t^d in two major categories: current accounts ∇S_t^d, loaning inside money market along with cash advances for interbank settlement; and lending to capital market $(1-\nabla)S_t^d$.

(6) $$S_t^d = \nabla S_t^d + (1-\nabla)S_t^d$$

ii. Return Rate for Depositors

The gross real gains of the bank on all its loans and advances (from money market and capital market) is denoted by E_b^{gT}, which it distributed to two kinds of deposit holders: First kind of deposit holders keep saving accounts and liquidate it on maturity, while second type deposit holders have time bound deposits and liquidate it before its maturity. The decision of second type deposit holder's pre-mature withdrawal (or who have current accounts) depends on their consumption dynamics and personal preferences.

If there is 'n' number of depositors than those loyal to the banks mainly functioning in the capital market are called loyal depositors (n – ω). These are the depositors who don't withdraw their time deposits from banks before maturity. On the other hand, the depositors who are mainly functioning in the money market are called fickle depositors (n- ώ). These are the depositors who withdraw money before the time of maturity (or have only current accounts in banks).

Let the total returns paid to those who withdraw their investment from money market before the time of maturity for consumption purposes or any other personal reasons is denoted by E_f^{gT}(returns for fickle deposit holders). Then, the returns paid to compensate and encourage the waiting for maturity agents, who are mainly time bound time deposits holders are denoted by E_l^{gT}(returns for loyal deposit holders). While returns from capital market investment in both private and government sector (which from now onward we will use interchangeably with capital market investment) is E_k^{gT}. To encourage the loyal deposit holders, the rate paid to them by bank consists of not only the returns on the time bound banks loans but also the portion of money market earnings deducted from the fickle deposit holders as ‘fine’. The investors at the bond market are initially deposit holders as per the assumption that all savings are deposited in banks and S_t^{GB} is equal to zero.

In Pakistan, the (fickle) depositors who withdraw their money before maturity are not fully compensated. The banks managers are tricky enough to cover up the damage caused by fickle depositors using different means, while the unpaid profits (to fickle depositors) are used for their own (banks) benefit and those of equity holders.

Returns from money market are incorporated as positive or negative shock, equal to reverse of inflation $P_t/P_{t+1} = \frac{1}{\jmath_t}$ in time period 1, while the total gross returns from capital market depends on the returns the banks get from private and public sector.

iii. Utility Vectors of Depositors (Both Loyal and Fickle depositors)

We presume the amount paid to each loyal depositor is equal to the total returns of the bank from capital market as bank profit is determined to be zero. The banks returns from capital market are equal to total deposits minus the pre-mature withdrawal $\left(\frac{(1+\jmath_t)A-(\nabla S_t^d)}{(1+\jmath_t)}\right)$ when multiplied by the bank’s lending rate for capital market (E_k^{gT}).

Where $'A'$the resource constraint for banks and their ‘total gross returns’ depends on it:

(7) $$A = \frac{(1-\nabla)S_t^d\left(1+E_k^{gT}\right)}{(1+\jmath_t)} + \frac{\nabla S_t^d}{(1+\jmath_t)}$$

$1+\jmath_t$ is inflation rate

Based on the above resource constraint (equation 7), returns to fickle and loyal depositor can be expressed respectively as follows:

(8) $$E_f^{gT} = \frac{\nabla S_t^d}{(1+\jmath_t)} E_b^{gT^8} \left\{\frac{1}{(n-\acute{\omega})}\right\}$$

(9) $$E_l^{gT} = \left\{\frac{(1+\jmath_t)A-(\nabla)S_t^d}{(1+\jmath_t)} E_k^{gT}\right\} \left\{\frac{1}{(n-\omega)}\right\}$$

The utility function (equation 8) for fickle depositors $(n-\acute{\omega})$ and rate paid to them (E_f^{gT}) is based on savings withdrawn before maturity ∇S_t^d. While, the utility function (equation 9) for

[8]It is to be noted that E_b^{gT} is exactly equal to the change in the value of saved money because of changes in the price level from one to the other period Pt/Pt+1

loyal depositors $(\mathrm{n}-\omega)$ and rate paid to them (E_l^{gT}) depends on 'resource constraint of banks' when short term current account savings ∇S_t^d are deduced from it.

iv. Depositor's Utility Functions Maximization

Total savings are equal to the total amount of money balances deposited in banks. The utility function of loyal and fickle depositors depend on the rate offered to them and on the sum of total savings (or resource constraint 'A') when tax is deduced from it. Besides, the utility maximization function for both 'loyal' and 'fickle' depositors not only depends on depositors risk aversion (ϖ/q) size but also on fraction constraint.

So the expected utility maximization vector for 'fickle depositors' (S_f^d), which the bank tries to maximize, can be denoted by the following interpretation:

$$\text{(10)} \qquad S_f^d = -(n-\acute{\omega})\left\{\frac{\{E_f^{gT}\}(1-\tau)\left[\frac{(1-\nabla)S_t^d\left(1+E_k^{gT}\right)}{(1+J_t)}+\frac{\nabla S_t^d}{(1+J_t)}\right]}{\varpi/q}\right\}^{-\varpi/q}$$

Similarly, the expected utility maximization vector for 'loyal depositors' (S_l^d) demonstrate the following contour.

$$\text{(11)} \qquad S_l^d = -(n-\omega)\left\{\frac{\{E_l^{gT}\}(1-\tau)\left[\frac{(1-\nabla)S_t^d\left(1+E_k^{gT}\right)}{(1+J_t)}+\frac{\nabla S_t^d}{(1+J_t)}\right]}{\varpi/q}\right\}^{-\varpi/q}$$

ϖ/q is Risk aversion (degree)

A. Utility Maximization when Loyal Depositors are paid only their Due Share

It is assumed that the utility of both type of deposit holders[9] are the same. As the higher returns which the loyal depositors get in comparison to fickle depositors is the price paid to them for a comparatively longer period of their consumption deferment.

So $$S_f^d = S_l^d$$

Replacing equation 10 and 11 in it

$$= -(n-\omega)\left\{\frac{\{E_l^{gT}\}^{-\varpi/q}(1-\tau)\left[\frac{(1-\nabla)S_t^d\left(1+E_k^{gT}\right)}{(1+J_t)}+\frac{\nabla S_t^d}{(1+J_t)}\right]}{\varpi/q}\right\}^{-\varpi/q}$$

Substituting in it the returns rate paid to fickle and loyal depositors (E_f^{gT}, E_l^{gT}) respectively from equation 8 and 9, and simplifying:

[9] who withdraw money before time and those who wait for maturity

$$-\frac{(\mathrm{n}-\acute{\omega})^{1+\varpi/\mathrm{q}}\{(\nabla S_t^d)\ \mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}\ \}^{-\varpi/\mathrm{q}}}{\varpi/\mathrm{q}} = -\frac{(\mathrm{n}-\omega)^{1+\varpi/\mathrm{q}}\left\{\left((1-\nabla)S_t^d\right)\{E_k^{gT}\}\right\}^{-\varpi/\mathrm{q}}}{\varpi/\mathrm{q}}$$

As the portion of total bank portfolio invested in capital market is $(1-\nabla)S_t^d$, therefore, we replace it with K_d^I (notation for capital produced) and the remaining amount of saving (∇S_t^d) as $1-\ K_d^I$.

Solving

$$\frac{(\mathrm{n}-\acute{\omega})^{1+\varpi/\mathrm{q}}\{(1-\ K_d^I)\ \mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}\ \}^{-\varpi/\mathrm{q}}}{\varpi/\mathrm{q}} = \frac{(\mathrm{n}-\omega)^{1+\varpi/\mathrm{q}}\{(\ K_d^I)\{E_k^{gT}\}\}^{-\varpi/\mathrm{q}}}{\varpi/\mathrm{q}}$$

As we know the E_k^{gT} is the capital return rate, we put its value from equation 5, where its degree is determined in the milieu of total investment (both public and private together) in capital market

Rearranging, we get

$$K_d^{I^{-\varpi/\mathrm{q}}} = \frac{(\mathrm{n}-\acute{\omega})^{1+\frac{\varpi}{\mathrm{q}}}\ (\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}})^{-\varpi/\mathrm{q}}}{(\mathrm{n}-\omega)^{1+\varpi/\mathrm{q}}\{\mathrm{P}(\Psi_{\sigma c})\}^{\varpi/\mathrm{q}}}\left\{\frac{1}{1+\left[\frac{(\mathrm{n}-\acute{\omega})^{1+\frac{\varpi}{\mathrm{q}}}\ (\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}})^{-\varpi/\mathrm{q}}}{(\mathrm{n}-\omega)^{1+\varpi/\mathrm{q}}\{\mathrm{P}(\Psi_{\sigma c})\}^{\varpi/\mathrm{q}}}\right]}\right\}$$

As $\frac{\varpi}{\mathrm{q}}$ is risk outline of house holders, it should be figured in better notation of single staging instead of its divisible characteristic. Therefore, we replace this term with a single notation of ß in the above equation, besides replacing K_d^I with $(1-\nabla)S_t^d$ as the total ventures taken in the capital market.

(12) $$(1-\nabla)S_t^d = \left[\frac{\mathrm{n}-\omega}{\mathrm{n}-\acute{\omega}}\right]^{1+1/ß}\ \mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}\{\mathrm{P}(\Psi_{\sigma c})\}\left\{\frac{1}{1+\left[\left[\frac{\mathrm{n}-\omega}{\mathrm{n}-\acute{\omega}}\right]^{1+ß}\ \mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}\{\mathrm{P}(\Psi_{\sigma c})\}\right]}\right\}$$

B. Utility Maximization for Depositors when Loyal Depositors Are Paid More than their Due Share as Reward for their Loyalty

In the second phase, we assume that the time and demand deposits are not managed separately. Instead, we take into consideration the total bank's portfolio (all deposits when $S_t^{GB}=0$) without a pre-tag of demand or time deposits.

we add an extra share equal to $\frac{\nabla S_t^d}{(1+Ꭻ_t)}\ \mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}$ in the returns of loyal depositors the bank offers them.

In such scenario, equation 8 for fickle depositors will remain the same; while equation 9 for loyal depositors will take the following shape (equation 9a):

(9a) $$\mathrm{E}_{\mathrm{l}}^{\mathrm{gT}*} = \left\{\frac{(1+Ꭻ_\mathrm{t})\mathrm{A}-(\nabla)\mathrm{S}_\mathrm{t}^\mathrm{d}}{(1+Ꭻ_\mathrm{t})}\mathrm{E}_{\mathrm{k}}^{\mathrm{gT}} + \frac{\nabla \mathrm{S}_\mathrm{t}^\mathrm{d}}{(1+Ꭻ_\mathrm{t})}\ \mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}\right\}\left\{\frac{1}{(\mathrm{n}-\omega)}\right\}$$

As discussed above, the utilities of those who prefer liquidation of the time deposits on maturity and those who withdraw money before the maturity time have the same dynamics; we put the utility functions of both the representative deposit holders as identity equation.

$$S_f^d = S_l^d$$

Repeating the same process of utility maximization with only one modification as

$$= -(\mathrm{n} - \omega)\left\{\frac{\{\mathrm{E}_l^{gT}\}(1-\tau)\left[\frac{(1-\nabla)\mathrm{S}_t^d}{(1+\jmath_t)} + \frac{\nabla\mathrm{S}_t^d\left(1+\mathrm{E}_b^{gT}\right)}{(1+\jmath_t)}\right]}{\varpi/q}\right\}^{-\varpi/q}$$

Substituting in it the returns rate paid to fickle and loyal depositors ($\mathrm{E}_f^{gT}, \mathrm{E}_l^{gT}$) respectively from equation 8 and 9a, and simplifying:

$$= -(\mathrm{n} - \omega)\left\{\left\{\frac{(1+\jmath_t)A - \nabla S_t^d}{(1+\jmath_t)} E_k^{gT} + \frac{\nabla S_t^d}{(1+\jmath_t)}\mathrm{E}_b^{gT}\right\}\left\{\frac{1}{(\mathrm{n}-\omega)}\right\}(1-\tau)\left[\frac{(1-\nabla)\mathrm{S}_t^d}{(1+\jmath_t)} + \frac{\nabla\mathrm{S}_t^d\left(1+\mathrm{E}_b^{gT}\right)}{(1+\jmath_t)}\right]\right\}^{-\frac{\varpi}{q}}$$

It is clear from the above equation that loyal depositor is bestowed not only with the returns from capital market $\frac{(1+\jmath_t)A - (\nabla)S_t^d}{(1+\jmath_t)} E_k^{gT}$ but also rewarded with portion $\frac{\nabla S_t^d}{(1+\jmath_t)} \mathrm{E}_b^{gT}$ from money market's earning.

Rearranging, cancelling out the inflation denominator from both sides of equation and simplifying exactly as above:

$$\frac{(\mathrm{n}-\acute{\omega})\left\{\frac{\nabla\mathrm{S}_t^d}{1}\mathrm{E}_b^{gT}\left\{\frac{1}{(\mathrm{n}-\acute{\omega})}\right\}\right\}^{-\varpi/q}}{\varpi/q} = -\frac{(\mathrm{n}-\omega)\left\{\left\{\frac{(1-\nabla)\mathrm{S}_t^d}{(1)}\mathrm{E}_k^{gT} + \frac{\nabla\mathrm{S}_t^d}{1}\mathrm{E}_b^{gT}\right\}\left\{\frac{1}{(\mathrm{n}-\omega)}\right\}\right\}^{-\varpi/q}}{\varpi/q}$$

Replacing and solving for total amount of investment in capital market K_d^I and rearranging we get

$$(13) \qquad (1-\nabla)\mathrm{S}_t^d = \left[\frac{\mathrm{n}-\omega}{\mathrm{n}-\acute{\omega}}\right]^{1+1/\beta} \frac{\mathrm{E}_b^{gT}}{\left\{\mathrm{P}(\Psi_{\sigma c}) - \mathrm{E}_b^{gT}\right\}}\left\{\frac{1}{1+\left[\frac{\mathrm{n}-\omega}{\mathrm{n}-\acute{\omega}}\right]^{1+1/\beta}\frac{\mathrm{E}_b^{gT}}{\left\{\mathrm{P}(\Psi_{\sigma c}) - \mathrm{E}_b^{gT}\right\}}}\right\}$$

Part 3: Government Constraint of Budget

To formulate the budget constraint of the government, we mainly take into account the budget likelihood (revenues and expenditure), seigniorages financing and government issued bonds. Total expenditure is considered as given and exogenously determined.

As we know that the inter-temporal budget constrained is but the government future flows and inflows estimates which guarantee the repayment of public money borrowed. In other words, the constraint's trajectory mainly accounts the budget inflows, making it equal to the account of present value.

i. Simple Budget Equation

To avoid the difference of value on the basis of changes in price level, we take debt and money balances in real terms.

(14) $$\frac{B_{D,t-1}}{P_{t-1}}\left(r_{f_{t-1}}\right) + G_t = T_t(Y_t) + \Delta\frac{B_{D,t}}{P_t} + \Delta\frac{B_{M,t}}{P_t}$$

$B_{D,t}$ is nominal debt at time period t, $B_{M,t}$ is nominal money balances at time period t, $B_{D,t-1}$ is nominal debt balance in previous time period, r_f is the risk free interest rate, , $Y_t$is aggregate income at time period t, G_t is total spending in time period t.

Replacing

$$\Delta\frac{B_{D,t}}{P_t} = \left[\frac{B_{D,t}}{P_t} - \frac{B_{D,t-1}}{P_{t-}1}\left(\frac{P_{t-1}}{P_t}\right)\right]$$

And substituting in reverse order recursively, we the following equation[10]:

(15) $$G_{t+1} - T_t(Y_t) + = \left[\frac{B_{D,t+1}}{P_{t+1}} - \frac{B_{D,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right] + \left[\frac{B_{M,t+1}}{P_{t+1}} - \frac{B_{M,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right] - \frac{B_{D,t}}{P_t}\left(r_{f_t}\right)$$

We assume that the present debt value doesn't reach a non-negative value no matter how long the time span extended over time period t. In other words, the rolling over of debt to next time period is a rule - condition where the ponzi scheme is a norm. In such scenario, the government prefers to be in the state of indebtedness constantly, forwarding the debt to next time period.

A very interesting feature of Pakistan economy can be brought about in this illustration. The government of Pakistan only tax three sectors of the economy, leaving the fourth (agriculture) sector as tax exempted. Following Chene (2006), and modifying for our model by assuming that agriculture production always accounts for a fixed portion of overall output, we divide the total taxable income in two portions:

1. The agriculture sector which is exempted from total tax Y_t^a and
2. The rest of the economy being taxed $\tau\,(Y_t - Y_t^a)$

We consider $T_t(Y_t) = \tau(Y_t - Y_t^a)$ being the total government revenues when agriculture sector is exempted from tax. In such scenario, the above originated government budget constrained (in equation 15) be devised as follows:

---

[10]Based on the above gesticulation, a more generalized budget constraint can be developed for more enlarged time periods.

(16) $G_{t+1} - \tau(Y_t - Y_t^a) = \left[\frac{B_{D,t+1}}{P_{t+1}} - \frac{B_{D,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right] + \left[\frac{B_{M,t+1}}{P_{t+1}} - \frac{B_{M,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right] - \frac{B_{D,t}}{P_t}(r_{f_t})$

Let $Y_t^a = e\,(Y_t)$

Then $T_{t+1}(Y_{t+1}) = \tau(Y_{t+1} - Y_{t+1}^a) = [\tau\,(1-e)Y_{t+1}]$

The agriculture sector accounts for a certain portion of total output in a country. The absence of tax imposition on agriculture sector reduce the expression $\tau(Y_{t+1} - Y_{t+1}^a)$ to $\tau(Y_{t+1} - (e)Y_{t+1})$ making the total output equal to $[(e)Y_t + (1-e)Y_t]$ and equation 16 as below:

(16a) $G_{t+1} - \tau(Y_{t+1} - eY_{t+1}) = \left[\frac{B_{D,t+1}}{P_{t+1}} - \frac{B_{D,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right] + \left[\frac{B_{M,t+1}}{P_{t+1}} - \frac{B_{M,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right] - \frac{B_{D,t}}{P_t}(r_{f_t})$

Before finding out a clear relationship between inflation and deficit budget dynamics, we have to put some light on the sustainability of the debt level. To discuss, the belief of Debt sustainability seems achievable, which narrates that at a point of time the government may honor the commitments it made over a period of time besides maintaining a balance budget. However, the real world scenario, where uncertainty is mostly a norm than odd, a very different story is displayed from the one articulated in the above exposition.

The introduction of uncertainty would make the model more realistic. However, before leading towards that end, it is customary to put light on the dynamics of public deficit financing. The individuals who purchase the risk free government securities in order to finance the budget deficit are basically renouncing their purchasing power in favor of government.

This purchasing power is supposedly a moral obligation on the part of the government to worth at least the same if not more, when returned back to the householders in future. So the government pays a positive interest rate in future to all the current time lenders as compensation to maintain their purchasing power. However, the uncertainty of future and debt sustainability in future is more attention seeking parameters to be addressed.

As in the above equations the interest rate as compensation has its incidence, however, market interest rate has insufficient application for future uncertainty and sustainability when seen in the tranversality perspective. Therefore, to avoid undue complexity and keep the model simple, we introduce the debt to money balances ratio (for replacing debt figures with money supplied) to avoid the debt related dynamics of the model, especially the debt sustainability.

Thus, deeming the debt to money ratio as constant and denoting it by Џ, we can get a value for debt balances in terms of money balances through simplification as below:

$$Џ = \frac{B_{D,t}}{B_{M.t}}$$

(16b) $B_{D,t} = Џ\, B_{M,t}$

Accommodating the debt balances in terms of money balances, the government budget constraint (equation 16) will take the following shape:

(17) $$G_{t+1} - \tau\{Y_{t+1} - (e)Y_{t+1}\} = \left[\text{ Џ}\frac{B_{M,t+1}}{P_{t+1}}\right] + \left[\frac{B_{M,t+1}}{P_{t+1}} - \frac{B_{M,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right]$$

As the economic literature argues, the budget expenditure (G_t) is linked with total output of the economy(Y_t). The total public expenditure is either less than or equal to it. Under some abnormal circumstances the public expenditure can also surpass the output level.

So lets $$G_t = \text{ց}\, Y_t$$

Where 'ց' can be less than one, greater than one or equal to one, signifying a level when expenditure is less than output, greater than output and equal to output, respectively. Also, expressing the seigniorages revenue[11] $\Delta\, B_{M,t}$ in real terms, the equation of budget constraint will take the shape as follows:

(18) $$Y_{t+1}(\text{ց}) - Y_{t+1}(\tau - e) = \left[\text{Џ}B_{m,t+1} + B_{m,t+1} - B_{m,t}\left(\frac{P_t}{P_{t+1}}\right)\right]$$

$$*\frac{\Delta\, B_{M,t}}{P_t} = \Delta\, B_{m,t}$$

We consider that money balances follow a steady state trend and grow at fixed rate. Then the fixed increase in money balances can be coated as below:

$$B_{m,t+1} - \text{ꝫ}B_{m,t} = B_{m,t}$$

$$B_{m,t+1} = B_{m,t}\,(1 + \text{ꝫ})$$

$$B_{m,t+1} = \emptyset\, B_{m,t}$$

The steady state economy assumption holds the grounds for output growth in the same fashion as for the money growth.

$$Y_{g,t+1} - \text{ꝫ}Y_{g,t} = Y_{g_t}$$

Let $$(1 + \text{ꝫ}) = \phi$$

$$Y_{gt+1} = \phi Y_{g_t}$$

Encompassing the fixed output and money growth considerations in budget constraint equation and replacing $\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}} = \frac{P_t}{P_{t+1}}$

$$Y_{t+1}(\text{ց} - \tau + e) = \left[\text{Џ}\emptyset B_{m,t} + \emptyset B_{m,t} - B_{m,t}\left(\frac{P_t}{P_{t+1}}\right)\right]$$

$$Y_{g_t}\,(\text{ց} - \tau + e) = B_{m,t}\left[\emptyset\,(1 + \text{Џ}) - \mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}\right]/\phi$$

[11] See Appendix III for seigniorage Notes

Assuming the money growth exactly equal to output growth, we replace the mathematical expression of change in money supply (∅) for change in aggregate perdition (ϕ).

$$(19) \qquad Y_{g_t}\,(g - \tau + e) = B_{m,t}\left[(1 + \text{Џ}) - \frac{E_b^{gT}}{\emptyset}\right]$$

Given that the level of expenditure and taxes are externally determined and E_b^{gT} is a positive or negative price shock, the above equation shows that the output depends on money balances $B_{m,t}$ and growth rate ∅. In the following segment we will determine the upshot for money balances and growth rate.

ii. Contriving 'Money Balances' and 'Growth Rate' for inserting in Budget constraint Equation

As the money balances $B_{m,t}$ and growth rate ∅ are endogenous in nature and its values are determined within the economic system, therefore in this segment we will set up the money balances and growth rate equations. Once the expressions for (a) money balances and (b) growth rate are mathematically determined, we will place it back in the above budget equation (17) for final treatment of our analysis.

A. Culmination of money Balances

To define the money balances in term of money demanded to finance transaction, we consider the current account cash into consideration. As we know from equation (6) that total stock of bank portfolio is divided into two mutually exclusive scraps, the one held by banks in the current account ∇S_t^d and the other invested in the capital market $(1 - \nabla)S_t^d$.

Money balance = total earning after tax * cash in hand[12]

$$(20) \qquad B_{m,t} = (1 - \tau)(1 - \Psi_{\sigma c})\,(\text{Ƥ})p_{KT}\left[\nabla S_t^d\right]$$

Where τ is taxes, $(1 - \Psi_{\sigma c})(\text{Ƥ})p_{KT}$ as previously defined in equation (4) is equal to total earning of the economy as all the householders are bestowed with one source of income (wages). While, the portion of total saving kept in current account for day to day transactions or the deposits liquidated before the time of maturity for consumption purposes is equal to (∇S_t^d).

As total earnings are equal to total savings and total savings are equal to total deposits, therefore S_t^d is equal to 1. Substituting in equation (6), the current account balances (∇S_t^d) are equal to the following expression

$$(21) \qquad \nabla S_t^d = 1 - (1 - \nabla S_t^d)$$

[12] depositors current account balances are included while their investment in bonds and capital markets is excluded

Taking the notation of $(1 - \nabla S_t^d)$ from the previously defined equation (13) of section II as follows:

$$(1 - \nabla)S_t^d = \left[\frac{n-\omega}{n-\acute{\omega}}\right]^{1+1/\beta} \frac{E_b^{gT}}{\{Ƥ(\Psi_{\sigma c}) - E_b^{gT}\}} \left\{\frac{1}{1+\left[\frac{n-\omega}{n-\acute{\omega}}\right]^{1+1/\beta} \frac{E_b^{gT}}{\{Ƥ(\Psi_{\sigma c}) - E_b^{gT}\}}}\right\}$$

Reducing the expression $\left[\left(\frac{n-\omega}{n-\acute{\omega}}\right)^{1+1/\beta} \frac{E_b^{gT}}{Ƥ\Psi_{\sigma c}-E_b^{gT}}\right]$ to be equal to $'\vartheta'$ as follow:

(21a) $$\vartheta = \left[\left(\frac{n-\omega}{n-\acute{\omega}}\right)^{1+1/\beta} \frac{E_b^{gT}}{Ƥ\Psi_{\sigma c}-E_b^{gT}}\right]$$

And then replacing this equation (21a) in equation 21, we get the following expression for current balances:

(22) $$\nabla S_t^d = 1 - (1 - \nabla S_t^d) = 1 - (\vartheta \frac{1}{1+\vartheta}) = \left(\frac{1+\vartheta-\vartheta}{1+\vartheta}\right) = \frac{1}{1+\vartheta}$$

Replacing equation (22) in the money balances equation (20), we get an expression as follows:

(23) $$B_{m,t} = [(1-\tau)(1-\Psi_{\sigma c})\,(Ƥ)p_{KT}]\frac{1}{1+\vartheta}$$

B. Evolution in Capital Stock (Output Growth Rate)

As the constant growth rate $\emptyset$ in this steady state model is based on Cobb Douglas production function, therefore, following Solow growth model (1956) with constant per capita growth we define our profit maximization function for representative firm as follows:

$$\pi_{max\,(p_K,h_l)} = Ƥ\left(\bar{g}_k^{1-\Psi_{\sigma c}}\, p_{kt}^{\Psi_{\sigma c}}\, h_{lt}^{1-\Psi_{\sigma c}}\right) - R_w\left(h_{lt}^{\Psi_{\sigma c}}\right) + k_r(p_{Kt})$$

During one-time period, the capital stock, the population and consequently the size of available work force remains the same. As the capital generation in the next time period depends on the portion of income saved by the households in current time period and invested in capital markets by banks.

Out of the Income invested in capital markets in time period t, the undertaken successful projects convert one unit of labor in the start of a time period t to one unit of capital by the end of that time period. In the Solow model, the stock of capital in time period t+1 depends on the amount of capital in the previous time period besides the amount of capital generated by the end of previous time period.

As we know that in our model, the portion of total bank portfolio invested in capital market is equal to $(1-\nabla)S_t^d$, therefore, the capital scenario in time period t exhibits the following illustration for capital accumulated in time period t+1

(24) $$p_{K_{t+1}} - p_{K_t}(1-\Upsilon) = (1-\nabla)S_t^d\left\{Ƥ\left(\bar{g}_k^{1-\Psi_{\sigma c}}\, p_{kt}^{\Psi_{\sigma c}}\, h_{lt}^{1-\Psi_{\sigma c}}\right)\right\}$$

Capital depreciates fully in one-time period and there is no lagged dependency. Therefore, the part of capital (Υ) depreciated in production process during the start and end time period t will become equal to one, and the portion $p_{Kt}\ (1-\ \Upsilon)$ becomes equal to zero. It is worth noting that the value of Υ reaches to 1 by the end of the year and therefore the time period (t) capital stock completely vanish away by the end of 't' time period. However, the capital complete deterioration takes a span of one year, it doesn't finish at the start of time period 't'.

Taking Υ equal to one, making the insured income stationary and defining the capital in the terms of per capita, the above equation will reduce to the following form:

(25) $$p_{K_{t+1}} = (1-\nabla)S_t^d\ \{\text{Ƥ}(\bar{g}_k^{1-\Psi_{\sigma c}}\ p_{kt}^{\Psi_{\sigma c}}\ h_{lt}^{1-\Psi_{\sigma c}})\}$$

We know from equation (4) that the total production in time period t based on the available resources (total capital) p_{KT} and given labour productivity Ƥ after the tax deduction is equal to:

$$(1-t)\{1-\Psi_{\sigma c}\}\ (\text{Ƥ})p_{KT}$$

Therefore

(26) $$\text{Ƥ}(\bar{g}_k^{1-\Psi_{\sigma c}}\ p_{kt}^{\Psi_{\sigma c}}\ h_{lt}^{1-\Psi_{\sigma c}}) = (1-t)\{1-\Psi_{\sigma c}\}\ (\text{Ƥ})p_{KT}$$

Replacing equation (26) in the above equation (25) we get the following expression for next time period capital:

(27) $$p_{K_{t+1}} = (1-\nabla)S_t^d\ (1-t)\{1-\Psi_{\sigma c}\}\ (\text{Ƥ})p_{KT}$$

By replacing $(1-\nabla)S_t^d = \vartheta \frac{1}{1+\vartheta}$ from previously defined equation (22) in the above equation (27), we get:

(27) $$p_{K_{t+1}} = \vartheta \frac{1}{1+\vartheta}\ (1-t)\{1-\Psi_{\sigma c}\}\ (\text{Ƥ})p_{KT}$$

We can acquire the gross rate of equilibrium 'capital growth' through dividing the future capital by the current period capital ($\emptyset = \frac{p_{Kt+1}}{p_{Kt}}$). Replacing the expression for $p_{Kt+1}$from equation (27), we can find the balance path of equilibrium growth rate as below:

$$\frac{p_{K_{t+1}}}{p_{K_t}} = (\vartheta \frac{1}{1+\ \vartheta})(1-t)\{1-\Psi_{\sigma c}\}\ (\text{Ƥ})p_{KT}/p_{KT} = \ \emptyset$$

(28) $$\vartheta = \frac{\vartheta(1-t)\{1-\Psi_{\sigma c}\}\ (\text{Ƥ})}{1+\ \vartheta}$$

Now finally getting back to the government budget constraint equation (19) and replacing the money balances ($B_{m,t}$) and change in capital stock (growth rate $\emptyset$) expressions in it from equation (23) and equation (28) respectively:

$$(g_{\flat} - \tau + e) = \left\{\frac{1}{1+\vartheta}(1-t)(1-\Psi_{\sigma c})\right\}\left[(1+Ц) - \frac{E_b^{gT}}{\frac{\vartheta(1-t)(1-\Psi_{\sigma c})(P)}{1+\vartheta}}\right]$$

Simplifying

(29) $$(g_{\flat} - \tau(1-e)) = \left\{\frac{1}{1+\vartheta}(1-t)(1-\Psi_{\sigma c})\right\}(1+Ц) - \left(\frac{E_b^{gT}}{\vartheta\ (P)}\right)$$

In addition to the revenues $\tau\{1 - e\}$ generated through imposing taxes on the total earning of all the inhabitants, the right hand side of budget equation (29) shows the added tax load needed to finance the government budget when there is a gap between total revenues and total expenditure. This portion of revenues is the segment which is to be finances by means other than direct taxation, i.e. seigniorages.

Inflation Analysis

Stating the economic situation of Pakistan economy, it is evident that the government tries to finance the ever increasing budget deficit through seigniorage revenues. As the decade long persistent inflation in Pakistan economy is perceived to be the result of enlarging fiscal deficits, therefore, we now check the implication of these kinds of fiscal moves (financing through seigniorages) in the perspective of deposit holders risk taking behavior.

We will analyze the response of inflation to change in 'growth prospects' and to 'seigniorage financing' of budget when the 'risk aversion' of deposit holders is more than zero.

A. Inflation Scenario When Fiscal tools focus Growth and Bank Depositors are not risk Takers

In this context we deal with two conditions. **First Case**, we take a situation where the tax revenues $\tau\{1 - e\}$ remain the same and the expenditure as proportion of total output ($g_{\flat}$) increases. In this case, we initially take into account the previously discussed concept of deposit rate where it is equivalent to the reverse of inflation, and finally the concept of deposit rate where it is expressed in terms of safe (no risk) rate (equation 52 of Part IV).

Second Case, the conventional view states that the debt issued by the government results in money formation when it is monetized by the central bank of the country, which in turn affects the supply/demand for money and increases the general price level. In this context, we will see the response of 'debt money ratio (Ц)' to change in 'budget expenditure' when risk aversion for deposit holders is positive.

i. First Case: The situation of no revenue changes while the expenditure as proportion of total output increases.

a. when the deposit rate (E_b^{gT}) is equal to just the reverse of inflation

If the deposit rate E_b^{gT} is equal to just the converse of inflation, we can find change in general price level due to change in expenditure as follow:

$$\text{(A5)} \qquad \left(\frac{E_b^{gT}}{\vartheta\,(P)}\right)(Ц + 1) = \tau\{1 - e\} - g + [(1 - t)\{1 - \Psi_{\sigma c}\}](Ц + 1)$$

$$\partial\frac{E_b^{gT}}{g} = -\frac{\vartheta\,(P)}{(Ц+1)} < 0$$

It is obvious from the above expression that budget financing will be inflationary in case the government relies on seigniorage.

b. <u>when the deposit rate (E_b^{gT}) is expressed in terms of safe (no risk) rate (R_p^g)</u>

As from deposit rate inference on the basis of funds allocation to both private and public projects and its illustration in terms of safe rates (equation 52), we have previously concluded that deposit rates are equal to:

$$E_b^{gT} = R_p^g\left[\frac{P_p^j\,(\sigma)}{\int} + 1\right] - (1 - P_p^j)\sigma$$

In such scenario, the less than zero change $\partial\frac{E_b^{gT}}{g} < 0$ points to a higher probability of successful projects (both of private and public sector). This in turn indicates higher growth prospects and boom in the economic activity, which will be discussed in details later in growth analysis segment.

ii. Second Case: Response of debt money ratio (Ц) to change in budget expenditure

Expressing the budget constraint equation in debt to money ratio (Ц) we get the following equation:

$$\text{(A7)} \qquad Ц = \frac{g - \tau\{1 - e\}}{\left[\{(1-t)(1-\Psi_{\sigma c})\} - \left(\frac{E_b^{gT}}{\vartheta\,(P)}\right)\right]} - 1$$

This can then be differentiated with respect to change in expenditure as follow:

$$\partial\frac{Ц}{g} = -\frac{\vartheta P}{E_b^{gT} + (1-t)(\vartheta P)(1-\Psi_{\sigma c})} < 0$$

The outcome is in line with the previous outcome (case 1) that budget deficit financing will be inflationary in case of seigniorage based financing of the budget. This can be readily verified from the above two situations when tax revenues remain the same, a change in budget expenditure is positively related to inflation while negatively related to debt to money balances ratio

A possible explanation can be from equation (16b) as debt to money ratio is indirectly related to money balances

$$B_{D,t} = Ц\, B_{M,t}$$

and money balances are directly related to inflation. Therefore, it seems quite obvious that the two outcomes are in contradiction. The outcome is obvious as debt issuance procedures follow a total opposite code in comparison to printing of money particulars. The government decision of moping out money from private saving pool results in households' purchasing power decrease and diversion of resources from private to public sector. So the general concept that the higher public debt leads to high level of inflation can be rejected.

iii. Inflation Dynamics when Public Expenditure is financed by Public Taxes

To divert from the economically hate loathed use of seiniorges for government budget financing instead of income taxation; we want to see its impact on inflation. To start, the economic phenomenon support a zero inflation outcome of a balanced budget as increase in public spending cause an equal reduction in private expenditure, leaving little room for any visible net change. The amount of money the public exchequer wants to spend is the amount of money no longer available to households and investors to squander.

It should be noted that the above argument is only in the context of net change in total spending. It doesn't refute the supremacy of public spending in comparison to private spending nor does it deny other disparities in welfare outcomes existed between the uses of private and public spending. To check the consistency of our analysis with the above mentioned economic phenomenon, we will now reverse the process. While keeping the public spending unchanged on the face of higher than zero risk aversion of deposit holders, we will first check the response of capital growth to change in taxes. And then, the response of inflation to changes in taxes.

A. The capital growth response change in taxes

We will differentiate capital growth equation with respect to tax, keeping the risk aversion of deposit holders greater than zero.

$$\emptyset = \frac{\vartheta(1-\tau)\{1-\Psi_{\sigma c}\}\,(\mathrm{P})}{1+\ \vartheta}$$

It can be eagerly verified that the change in output growth (equal to capital growth) due to change in taxes τ is less than zero $(\partial\frac{\emptyset}{\tau} < 0)$[13]. This outcome is against most of the existing studies outcomes on the subject. The outcome indicates the suppression of growth in case of additional tax imposition, leaving little room for private spending (which insures more efficient use of resources). The argument of Vickrey (1960) outcomes fit for our result where he stated that there is no virtue of supremacy attached with balance budget practices, except that it can be considered a bench mark.

---

13 $-\frac{\mathrm{P}\vartheta(1-\Psi_{\sigma c})}{1+\vartheta}$

It is further emphasized that balance budget can be inappropriate with the notion of full employment. Moreover, if the total saving to income ratios show a comparatively lower figure, than it may result in a depressed level of output growth, boosting the need of government intervention for required capital accumulation.

B. The inflation response change in taxes

Again, the economic literature varies on the issue of inflationary and deflationary outcome of a balanced budget.

To discuss, higher rate of income taxes can be inflationary because of the substitution effect where the workers demand higher wages[14]. Moreover, the higher rates of taxation can trigger up the cost of production, and may result in cost push kind of inflationary outcomes. On the other hand, direct taxation negatively affects householders' consumption and investors' saving and consequently the investment and capital accumulation, leading to deflation in the economy. Further, even the deflationary outcome of higher taxation varies in its nature. An example can be quoted from Vichrey (1960) that some direct taxation, like higher taxing rates for gift and estate can be non-deflationary, as response of general price level to such taxation spread over a very longer period of time.

Without indulging further in the theoretical aspects of income taxation, we want to see the impact of changes in direct taxes on inflation, keeping the public spending unchanged and risk aversion of deposit holders greater than zero. For this reason, we will first put the value of capital determinant ϑ (from equation 21a) in the equation capital growth / output growth (equation 28) and solve it for the deposit rate E_b^{gT} (inverse of inflation).

(A12)
$$E_b^{gT} = \frac{\emptyset(1+\vartheta)(\mathrm{P}\Psi_{\sigma c})}{\left[\frac{n-\omega}{n-\acute{\omega}}\right]^{1+1/\beta}(1-t)\{1-\Psi_{\sigma c}\}(\mathrm{P})} + 1$$

Once it is expressed in deposit rate as above (equation A12), it can be fervently verified that the change in inflation due to change in taxes τ is negative $(\partial\frac{E_b^{gT}}{\tau} > 0)$ as E_b^{gT} is the converse of inflation.

CONCLUSION

Excessive claims on government in developing economies leads to multiple economic ailments like crowding out, inflation and monetary expansions. Both recurring governments efforts and IMF structural adjustments programs seems futile exercises in reducing the bourgeoning fiscal deficits of these developing economies. As in case of Pakistan, the persistence increases in the fiscal deficit over the last three decades not only diluted the real sector performance, but also

---

[14] Workers demand higher wages to compensate for the loss of utility due to soaring inflation.

negatively affected the balance of payments strains, causing inflationary pressures. Despite multiple attempts and recurring efforts on the part of monetary authorities to tilt down the fiscal mess in the economy, no worth noticing success is witnessed. Moreover, fiscal authorities determine the targets of taxes and level of expenditure without respecting the inter-temporal solvency conditions. These evidences and utter failure of monetary policy in maintaining general price level in specified limits display enough evidence for fiscal dominancy (non-Ricardian regime) in the economy of Pakistan. In other words, the unsustainable fiscal deficits are often being responsible for unhealthy monetary expansion and consequent increase in inflation. Therefore, it is emphasized that the dominant challenge for fiscal and monetary managers is the need for efficient policy coordination to achieve appropriate balance between stable inflation, equitable income distribution and more importantly the diminution of inflation. However, there is hardly any thorough study on the subject in Pakistan except a few trifling contributions.

Thus, in this study we have tried to see the impact of fiscal and monetary policies together for gauging the impact on inflation for developing economies in general and for Pakistan in Particular. We have tried to bridge the two widen poles of monetary and fiscal endogenous inflation dynamics. The interactions between the two frames are judged and premeditated while keeping the financial intermediaries as the main invigorator in the analysis. Economic literature is evident that soaring inflation prospects depends on the devise of banks loans to different projects, besides budget expenditure, public debt and seigniorages. The decisions of banks for allocating loans to risky ventures principally depend on the risk taking behavior of savers. Thus, this study includes the willingness of depositors for taking risk, allowing banks to invest in more risky ventures. The study put light on inflation prospects while first changing the expenditure level (keeping taxes unchanged) and then the taxes level (keeping expenditure unchanged) on the face of low and high risk aversion of depositors.

The study further analysis that the response of general price level to seigniorages, and confirmed that such financing is inflationary in nature. On the other hand, debt financing is not inflationary as debt issuance procedures follow a totally converse code in comparison to printing money particulars. Divergence of resources from private to public sector reduces the availability of funds for private investors and decreases the purchasing power of households. This result is opposite to the normally established outcome that higher public debt level can lead to high level of inflation. As for a given level of lower general price level in developing economies, any increase in expenditure, keeping the revenue unchanged, will shift cash in hands to capital accretion.

In other words, we can say that if the existing level of inflation is low, decisions of expansionary policies on the part of fiscal authorities can be fruitful in channelizing the money held by the householders to investment. The added ingress in the circuits of investment pools initiate further capital accumulation. Whereas examining finally the impact of changes in general price level on capital growth '∅,' the study concludes that a higher inflation will guides to lower than otherwise expected growth and capital accumulation. The inference is in line with other studies on the subject and the economic theory supports the argument on the basis of higher demand for real balances when general price level is high

REFERENCES

Acemoglu, D., & Restrepo, P. (2022). Demographics and automation. *The Review of Economic Studies*, *89*(1), 1-44.

Adrian, T., & Shin, H. S. (2020). Liquidity and leverage. *Journal of Financial Intermediation, 41*, 100-112.

Agha, A. I., & Khan, M. S. (2006). An empirical analysis of fiscal imbalances and inflation in Pakistan. *SBP Research Bulletin*, *2*(2), 343-362.

Ajello, A. (2016). Financial intermediation, investment dynamics, and business cycle fluctuations. *American Economic Review, 106*(8), 2256–2303.

Azariadis, C., & Smith, B. D. (1996). Private information, money, and growth: Indeterminacy, fluctuations, and the Mundell-Tobin effect. *Journal of Economic Growth, 1*(3), 309-332.

Barro, R. J. (1990). Government spending in a simple model of endogenous growth. *Journal of Political Economy*, *98*(5, Part 2), S103-S125.

Begenau, J. (2019). Capital requirements, risk choice, and liquidity provision in a business-cycle model. *Journal of Financial Economics, 134*(1), 1-22.

Bencivenga, V. R., & Smith, B. D. (1993). Some consequences of credit rationing in an endogenous growth model. *Journal of Economic Dynamics and Control, 17*(1-2), 97-122.

Brunnermeier, M. K., & Sannikov, Y. (2021). The I theory of money. *The Review of Economic Studies*, *88*(3), 1068-1108.

Chen, B. L. (2006). Economic growth with an optimal public spending composition. *Oxford Economic Papers*, *58*(1), 123-136.

Diamond, D. W., & Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, *91*(3), 401-419.

Farhi, E., & Tirole, J. (2015). Liquid bundles. *Journal of Economic Theory, 158*, 634–655.

Greenwood, R., Hanson, S. G., & Stein, J. C. (2020). A comparative-advantage approach to government debt maturity. *The Journal of Finance*, *75*(1), 1-50.

Guerrieri, V., & Shimer, R. (2014). Dynamic adverse selection: A theory of illiquidity, fire sales, and flight to quality. *American Economic Review*, *104*(7), 1875–1908.

He, Z., & Krishnamurthy, A. (2013). Intermediary asset pricing. *American Economic Review, 103*(2), 732–770.

Hung, F. S. (2003). Inflation, financial development, and economic growth. *International Review of Economics & Finance*, *12*(1), 45-67.

Jones, L. E., & Manuelli, R. E. (1995). Growth and the effects of inflation. *Journal of Economic Dynamics and Control, 19*(8), 1405-1428.

Kaplan, G., Moll, B., & Violante, G. L. (2020). The great lockdown and the big stimulus: Tracing the pandemic possibility frontier for the U.S. *Journal of Economic Perspectives*, *34*(4), 41-60.

Kiyotaki, N., & Moore, J. (2019). Liquidity, business cycles, and monetary policy. *Journal of Political Economy*, *127*(6), 2926-2966.

Krishnamurthy, A., & Vissing-Jorgensen, A. (2022). The aggregate demand for Treasury debt. *Journal of Political Economy*, *130*(1), 1-45.

Kurlat, P. (2013). Lemons markets and the transmission of aggregate shocks. *American Economic Review*, *103*(4), 1463–1489.

Maggiori, M. (2017). Financial intermediation, international risk sharing, and reserve currencies. *American Economic Review*, *107*(10), 3038–3071.

Reis, R. (2021). Losing the inflation anchor. *Brookings Papers on Economic Activity*, *2021*(1), 307-361.

Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy*, *94*(5), 1002-1037.

Schreft, S. L., & Smith, B. D. (1997). Money, banking, and capital formation. *Journal of Economic Theory*, *73*(1), 157-182.

Schularick, M., & Taylor, A. M. (2021). Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870–2008. *American Economic Review*, *111*(4), 1029-1069.

Stroebel, J. (2016). Asymmetric information about collateral values. *The Journal of Finance*, *71*(3), 1071–1112.

Vickrey, W. S. (1960). The Balanced Budget: Defense against Inflation? *Challenge*, *8*(7), 57-61.

Worrell, M. D. (2000). Monetary and fiscal coordination in small open economies (No. 0-56). *International Monetary Fund*.