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Growth Equilibria: Policy-Driven Capital Accumulation and Pakistan's Economic Destiny

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ABSTRACT

The transmission mechanism of fiscal and monetary policy depends much on financial intermediaries, who also shape economic development and production. In developing nations like Pakistan, sector-specific tax exemptions and fiscal supremacy are somewhat common. This study underlines the important part of intermediaries in policy coordination by directly simulating the development response in such surroundings. It shows how loan-advancing techniques of the banking sector and the risk-taking behavior of deposit holders affect their impact on long-term development paths and productive capacity.

INTRODUCTION

Fiscal and monetary policy are the main tools for reaching macroeconomic goals including steady output increase. Although both policies have different areas of influence, their effects are linked and underlines the importance of effective policy coordination to support high and sustainable development. Policies that lack coordination may lose their potency, so failing to meet development aims. For example, too large fiscal deficits meant to boost total demand might cause exchange rate volatility and compromise the balance of payments, therefore upsetting long-term economic stability.

When fiscal policy rules, marginalizing monetary measures, macroeconomic instability gets more severe, which increases financial uncertainty and discouragement of wise investments. Fiscal activism typically eclipses monetary policy in emerging nations with limited central bank autonomy, as Worrell (2000) notes, therefore forcing central banks to fund public sector deficits, often at the expense of long-term development.

Studies by Barro (1990) and Jones and Manuelli (1995) show how greatly government fiscal policies impact output dynamics. Although the literature looks at how public policies affect development, it sometimes ignores the crucial part financial intermediary's play. However, their inclusion in growth

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models is essential since financial intermediaries are very necessary for guiding money into profitable ventures and affecting capital creation.

This paper includes the banking industry in order to investigate how monetary and fiscal policy together affect production growth. Apart from enabling financial transactions, banks also help to mediate investment decisions, therefore affecting the distribution of funds across initiatives with different risk and return profiles. Explaining development patterns depends on knowing how policies interact with banking tactics like loan allocation and liquidity management.

We construct our framework using fundamental models. Originally developed from Diamond and Dybvig (1983), the idea of fickle depositors helps evaluate how banking contracts could minimize the negative consequences of early deposit withdrawals. We use Bencivenga and Smith (1993) for endogenous growth analysis to show how financial intermediaries convert savings into productive capital thereby promoting development.

Schreft & Smith's (1997) model is pertinent for how banks provide liquidity into the system and how government borrowing could drown out private investment, therefore affecting long-term development paths. From the production standpoint, Romer (1986) provides our framework since externalities in the manufacturing process help to sustainably increase growth.

Examining this literature, we created an overlapping generations (OLG) model fit for a developing nation such as Pakistan. The model catches the interaction between dynamics of fiscal policy and household portfolio choices. While the government's expenditures are, exogeniously defined and funded by direct taxation and bond issuing, households choose between keeping cash and investing in illiquid assets.

Through the prism of financial intermediaries, the model clarifies how the banking sector's portfolio strategy shapes economic prospects not only by public expenditure but also by means of production reactions to fiscal shocks. The analysis questions received knowledge and exposes, for example that, depending on how banks distribute money, government expenditure related to significant deficits can boost development. This study essentially investigates the complex interactions of fiscal and monetary policy, financial intermediaries, and output growth, therefore providing novel understanding of the processes behind sustainable development in emerging countries. The results suggest fresh directions for policy debates, particularly with relation to the best coordination of public financing and banking practices to optimize development possibility.

Motivation of the Study

Underutilized financial intermediation and poor policy coordination often aggravate the ongoing growth difficulties developing economies like Pakistan face. Although both monetary and fiscal policies seek to boost output, their impact relies on the way financial intermediaries distribute resources. As a major actor in the distribution of capital, the banking industry shapes the course of economic development by means of its portfolio decisions and reaction to policy changes. Still, the theoretical literature lacks a coherent mathematical framework including financial intermediation into growth modeling. This work fills up that void by developing a theoretical model capturing the relationship between output growth via financial intermediaries and policy dynamics.

Conceptual and Theoretical Framework

Grounded in Romer's (1986) endogenous growth theory, the study builds an OLG model then expands it using Diamond & Dybvig's (1983) banking behavior paradigm. The model shows how the risk preferences of depositors affect the lending policies of banks, therefore affecting the accumulation of productive capital. The model investigates several equilibrium routes by modeling shocks in fiscal and monetary policies, therefore highlighting how policy coordination could either improve or limit long-term output growth. This theoretical approach offers insightful analysis of the processes either promoting or hindering sustainable development in underdeveloped countries.

Research Objectives

- To formulate a mathematical model that reflects the interaction between fiscal and monetary policy through financial intermediaries, emphasizing output growth dynamics utilizing overlapping generations (OLG) and endogenous growth models
- To investigate the risk preferences of depositors affect banks' capital allocation choices and thereby affect output growth.
- To offer theoretical insights into the structural impediments to growth in emerging economies, and on how coordinated policies could hasten output increase.

LITERATURE REVIEW

Theoretical research of output growth has been much advanced. Emphasizing the part knowledge spills and capital accumulation play in promoting long-term development, Romer (1986) first presented the endogenous growth theory. While Diamond and Dybvig (1983) theoretically analyzed banks behavior in response to liquidity demands, Barro (1990) and Jones & Manuelli (1995) showed how government spending effects economic trajectories. Bencivenga and Smith (1993) expanded this by demonstrating how reallocating savings to profitable investments improves financial intermediaries' ability to spur development.

More recent research has enhanced this knowledge. Developing models examining the link between policy coordination and sustainable development were Bolhuis, Koosakul, and Shenai (2024). Sheard (2023) underlined how financial middlemen help to stabilize capital allocation. Analyzed how leverage cycles and bank risk-taking practices impact development outcomes were Adrian and Shin (2020) and Brunnermeier and Sannikov (2021). While Hung (2003) showed the growth-limiting implications of policy misalignment, Agha & Khan (2006) offered a framework for comprehending the link between fiscal policy and output in the context of developing economies.

Research Gap

Though a lot of research has been done on growth dynamics, few studies specifically use a strictly mathematical approach to explicitly analyze the function of financial intermediaries in output determination. Understanding how banks moderate growth is crucial in Pakistan, because fiscal supremacy might skew capital flows. There is a great void left by the lack of theoretical models that reflect the effect of depositor risk behavior on resource allocation and output. This work closes that gap by constructing a thorough theoretical model simulating output growth results under several policy environments.

THE MODEL

Designed to fit Pakistan's particular economic situation, the model is based from a two-generation viewpoint. Considering all people as the workforce, each one of them is operating as a deposit holder. Workers with different skill levels are not distinguished from one another. Rather, individuals who liquidate their accounts early differ from others who are ready to wait for the maturity of time deposits. Further classification of depositors based on their risk preferences—low or high risk aversion—is done. The model is built in three sections, each with several segments and sub-segments that lead toward knowledge of output dynamics.

With three main sections—output (equation 1), firms (equation 3), wage rates (equation 4), and capital returns (equation 5)—part 1 of the model provides the basis for output determination. Establishing the capital growth equation depends on the production function (equation 1); the expression for capital returns (equation 5) is applied in depositors' utility maximizing vectors (equations 12 and 13). Important for understanding long-term output expansion, wage rates (equation 4) flow into the capital growth equation (equation 26).

Comprising four sections, Part 2 of the model centers on the contribution of the banking industry to production growth. It begins with the bank's distribution of entire deposits among several businesses (bonds, money, and capital markets) satisfying required reserve criteria (equation 6). Here, the main objective is to establish deposit rates for many kinds of deposit holders. Equations 8 and 9 let the model derive deposit rates given to capital market and money market lenders and provide exclusive rates for erratic and faithful depositors. Since banks distribute a sizable amount of deposits to the capital market, capital market performance mostly determines deposit rates. From the capital market view, the model then maximizes depositors' utility functions (equations 12 and 13), which become crucial in comprehending how banking practices affect capital formation and, hence, output growth.

Third element of the model creates a budget constraint equation to examine development results. Using government debt, taxes, and public spending (equation 14), segment 1 creates a basic budget equation. Equations 16 and 16a let one account for tax systems unique to Pakistan's economy. Segment 2 is on developing formulas for rates of capital growth. Determination of growth rate depends on total production and wage rate equations (equations 1 and 4 from Part I) together with capital market investments (equation 22 from Part III). These growth rate expressions are replaced back into the final government budget constraint calculation (equation 19) to evaluate how long-term growth is shaped by fiscal policy, banking portfolio decisions, and capital market returns taken all together.

All told, the model shows how closely fiscal policy, banking sector dynamics, and capital returns interact to produce sustainable output growth. It offers a complete framework to examine development paths in emerging nations such as Pakistan by combining the behavior of public sector actions and financial intermediaries.

Output, Firms, Capital Returns and Wage Rate

i. Output

Regarding production, the economy is regarded as one good and businesses compete in a totally competitive setting. While the whole output depends on the "private" money allotted to the representative company and the infrastructure assistance given by public ventures, the output of the representative firm is equal to Y_t. The degree of output rises in cases when the success rate of public and private initiatives launched is high. This emphasizes the need of effective public investment and capital allocation in increasing general economic development.

(1) $$Y_t = f\ \left(g_k\, p_k\, h_l\right) = \text{Ƥ}\left(\bar{g}_k^{1-\mathrm{N}_{\sigma c}}\, p_{kt}^{\mathrm{N}_{\sigma c}}\, h_{lt}^{1-\mathrm{N}_{\sigma c}}\right)$$

$\bar{g}$ is public capital, p_{Kt} is Private Capital in time period t and h_{lt} is Labor in time period t.

As each worker works for set hours, the model provides no leisure-work trade off dimensions. Assumed to be higher than zero, a certain fraction of $\bar{g}_k$ depends on private savings transferred to public sector initiatives by the financial intermediaries as investment. It should be mentioned that $\bar{g}$ relates to public investment for infrastructure connected to the manufacturing sector. Apart from that, it offers favorable spill-over effects and the environment fit for developing the national production sector. Two mutually exclusive poles of consumption and savings divide the whole profits. Every unit saved is equal to one unit of consumption sacrificed for investment, so producing one unit of capital in the following time period.

ii. Firms

There is perfect competition; businesses are price conscious and aim to maximize profit. The whole cost of the businesses covers salaries paid and rent paid. The companies' income is determined by the volume of output they create and market within one time span. The company makes no profit; its overall income matches its expenses. With a truly competitive market, labor supply in the economy is fully utilized and corporate marginal expenses match marginal profits.

The salaries of workers and rent the capital assets pay back define total earning in the economy. Equation 1 will thus shown to be as follows:

(2) $$Y_t = f\ \left(g_k\, p_k\, h_l\right) = \text{Ƥ}\left(p_{Kt}^{\mathrm{N}_{\sigma c}}\, h_{lt}^{1-\mathrm{N}_{\sigma c}}\right)$$

It can be made more specific by taking the derivation of firm's profit with respect to 'capital' (p_{Kt}) and 'labour' $\left(h_{lt}^{N_{\sigma c}}\right)$.

(3) $$\text{Ƥ}\, F(\bar{g}_k\, p_{Kt}\, h_{lt}) = \mathrm{R}_w\left(h_{lt}^{\mathrm{N}_{\sigma c}}\right) + \mathrm{k}_r(p_{Kt})$$

iii. *Capital Returns and Wage Rate*

Wage rate R_w and Capital return K_r are given below in equation 4 and 5 by taking the derivatives of equation (1) with respect to labor and capital, respectively we get

$$\mathrm{R}_w = \{1 - \mathrm{N}_{\sigma c}\}\,(\mathrm{Ƥ})\,\frac{p_{kt}^{\mathrm{N}_{\sigma c}} / h_{lt}^{\mathrm{N}_{\sigma c}}}{\bar{g}_k^{\mathrm{N}_{\sigma c}-1}}$$

Labor supply is inelastic, and labor force is fully utilized ($h_{lt} = 1$).

As public investment is infrastructure related and part of total investment, therefore, total capital is equal to both private and public capital assortment[5]. Further, the model deem public capital[6] ($\bar{g}_k$) equal to the private investment p_{kt}. Based on these assumptions, we can further simplify wage rate as follows:

$$\{1 - \mathrm{N}_{\sigma c}\}\,(\mathrm{Ƥ})\,\frac{p_{kt}^{\mathrm{N}_{\sigma c}+1-\mathrm{N}_{\sigma c}}}{h_{lt}^{\mathrm{N}_{\sigma c}}}$$

(4) $$\mathrm{R}_w = \{1 - \mathrm{N}_{\sigma c}\}\,(\mathrm{Ƥ})\,p_{KT}$$

R_w = Real wage Rate

In the same fashion we can find the capital rent as follows:

$$\mathrm{k}_r = \mathrm{N}_{\sigma c}\,\mathrm{Ƥ}\,\frac{\bar{g}_k^{1-\mathrm{N}_{\sigma c}}}{z_{kl}^{\mathrm{N}_{\sigma c}}}$$

$$= \mathrm{N}_{\sigma c}\,\mathrm{Ƥ}\,\frac{p_{kt}^{1-\mathrm{N}_{\sigma c}}}{z_{kl}^{\mathrm{N}_{\sigma c}}}$$

(5) $$\mathrm{k}_r = \mathrm{Ƥ}(N_{\sigma c}) = \mathrm{E}_{\mathrm{k}}^{\mathrm{gT}}$$

K_r is Real return on capital, while $z_{kl}^{N_{\sigma c}}$ is Capital labor ratio ($p_{Kt}^{N_{\sigma c}} / h_{lt}^{N_{\sigma c}}$)

Savings as Deposits, Returns on Total Saving, Allocations by Banks and Utility of Deposits for Individuals Bank portfolio

i. *Total savings and deposits distribution*

By defining various kinds of returns, the factual elements have been given a more reflecting connotation and help to form the model more in line with the modern banking structure. While saving possibilities are more complicated and the division and pattern of banking activities in Pakistani economy are somewhat unique in several ways, When it comes to population classification on really financial grounds, Pakistani society is split in two more general terms. Unskilled or low income groups, whose main saving brands are essentially defined as money and demand deposits; and well-heeled class or skilled group, who are either not interested in

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[5] $\bar{g}_{kt} + p_{kt} = p_{Kt}$

[6] a spread out externality in the production function

equity market due of religious reasons or doesn't want to coup up with such an equity market where the desired sophistication is missing.

For minimalistic reasons, however, total public savings are split into two main categories: money maintained as bank deposits and savings made for the purchase of government bonds. Both approaches are absolutely vital for precisely calculating the interest rates—that is, returns on savings—that the banks provided to consumers.

Bonds are government issued certificates bearing (usually) a fixed interest rate, issued by government to be used for funding of budget deficit. While banks reserve a portion S_r^r in cash to fulfill the CRR requirements, they invest entire deposits in the capital market and money market. The capital investment is supposed to be time bound deposits; money market investments are tacitly intended to be time unbound demand deposits. For the purchase of government bonds, banks also commit a specific fraction '$b^{g'}$ of the overall deposits.

$$S_t^{GB} = b^g(S_t^d).$$

Under such circumstances, the total loan-able bank's portfolio B_p^l is equal to

$B_p^l = S_t^d\,(1 - \;S_t^{GB} - S_r^r)$

Where, B_p^l is the loan-able portfolio that the bank can lend after S_r^r (cash reserve requirement) and S_t^{GB}(credit used for the purchase of bonds) are deduced from the aggregate total savings (S_t^d).

We take up total loan-able bank's portfolios (B_p^l) to be initially equal to aggregate savings S_t^d, as there is no pre-specified allocation of bank portfolio for bonds purchase and the reserve requirement S_r^r is negligible.

Bank divides its total deposits S_t^d in two major categories: current accounts $\bowtie S_t^d$, loaning inside money market along with cash advances for interbank settlement; and lending to capital market $(1-\bowtie)S_t^d$.

(6) $$S_t^d = \bowtie S_t^d \;+\; (1-\bowtie)S_t^d$$

i. Return Rate for Depositors

E_b^{gT} marks the gross real gains of the bank on all its loans and advances (from money market and capital market) which it dispersed to two types of deposit holders: While second type deposit holders have time constrained deposits and liquidate them before their maturity, first sort of deposit holders maintain saving accounts and liquidate them at maturity. Depending on their consumption patterns and personal preferences, second type deposit holders—who have current accounts—decide on their pre-mature withdrawal.

Loyal depositors (n – ɵ) are those who mostly operate in the capital market and have less number than those who are dependable for the banks. These are the depositors whose time deposits they do not withdraw from banks before maturity. Conversely, fickle depositors (n-ώ) are the ones mostly engaged in the money market. These are the depositors—that is, those who have either merely current accounts in banks or remove money before the maturity date. E_f^{gT} (returns for fickle deposit holders) denotes let the total returns paid to individuals who remove their investment from money market either for consuming needs or any other personal reasons. The returns then go toward compensating and motivating the waiting for maturity agents—mostly time bound time deposits holders—are symbolized by E_l^{gT}. From now on we shall use interchangeably with capital market investment E_k^{gT}, albeit returns from capital market investment in both private and government sector. The rate provided to the devoted deposit holders by the bank comprises not only the returns on the time bound banks loans but also the percentage of money market gains removed from the fickle deposit holders as "fine". Originally deposit holders, the bond market investors assume all funds are deposited in banks and S_t^{GB} is equal to zero.

The erratic depositors in Pakistan who take their money out before maturity are not fully reimbursed. While the unpaid profits (to fickle depositors) are used for their own (banks) gain and those of equity holders, the banks managers are cunning enough to compensate the loss created by fickle depositors using alternative means.While the total gross returns from capital market depend on the returns the banks obtain from both private and public sector, returns from money markets are incorporated as positive or negative shock, equivalent to reverse of inflation $P_t/P_{t+1} \; = \frac{1}{F_t}$

ii. ***Utility Vectors of Depositors (Both Loyal and Fickle depositors)***

Since bank profit is shown to be zero, we assume the amount given to each loyal depositor is equal to the total returns of the bank from capital market. The banks' capital market returns equal their total deposits less the pre-mature withdrawal $\left(\frac{(1+F_t)A-(\bowtie S_t^d)}{(1+F_t)}\right)$ when multiplied by the bank's lending rate for capital market (E_k^{gT}).

Where $'A'$the resource constraint for banks and their 'total gross returns' depends on it:

$$A = \frac{(1-\bowtie)S_t^d\left(1+E_k^{gT}\right)}{(1+F_t)} + \frac{\bowtie S_t^d}{(1+F_t)} \tag{7}$$

$1 + F_t$ is inflation rate

Based on the above resource constraint (equation 7), returns to fickle and loyal depositor can be expressed respectively as follows:

(8) $$E_f^{gT} = \frac{\bowtie S_t^d}{(1+F_t)} E_b^{gT} \left\{\frac{1}{(n-\hbar)}\right\}$$ [7]

(9) $$E_l^{gT} = \left\{\frac{(1+F_t)A-(\bowtie)S_t^d}{(1+F_t)} E_k^{gT}\right\} \left\{\frac{1}{(n-\text{e})}\right\}$$

The utility function (equation 8) for fickle depositors $(n-\hbar)$ and rate paid to them (E_f^{gT}) is based on savings withdrawn before maturity $\bowtie S_t^d$. While, the utility function (equation 9) for loyal depositors $(n-\text{e})$ and rate paid to them (E_l^{gT}) depends on 'resource constraint of banks' when short term current account savings $\bowtie S_t^d$ are deduced from it.

iii. Depositor's Utility Functions Maximization

Total savings are equal to the total amount of money balances deposited in banks. The utility function of loyal and fickle depositors depend on the rate offered to them and on the sum of total savings (or resource constraint 'A') when tax is deduced from it. Besides, the utility maximization function for both 'loyal' and 'fickle' depositors not only depends on depositors risk aversion $(R/\forall)$ size but also on fraction constraint.

So the expected utility maximization vector for 'fickle depositors'(S_f^d), which the bank tries to maximize, can be denoted by the following interpretation:

The whole quantity of money balances placed in banks is exactly what total savings are. The rate given to faithful and erratic depositors determines their utility function as well as the overall savings (or resource constraint "A") upon tax deduction from them. Apart from depositor risk aversion $(R/\forall)$ size, the utility maximizing function for both "loyal" and "fickle" depositors depends also on fraction constraint.

Thus, the following describes the expected utility maximizing vector for "fickle depositors," (S_f^d), which the bank seeks to maximize:

(10) $$S_f^d = -(n-\hbar)\left\{\frac{\{E_f^{gT}\}(1-T)\left[\frac{(1-\bowtie)S_t^d\left(1+E_k^{gT}\right)}{(1+F_t)}+\frac{\bowtie S_t^d}{(1+F_t)}\right]}{R/\forall}\right\}^{-R/\forall}$$

Similarly, the expected utility maximization vector for 'loyal depositors' (S_l^d) demonstrate the following contour.

(11) $$S_l^d = -(n-\text{e})\left\{\frac{\{E_l^{gT}\}(1-T)\left[\frac{(1-\bowtie)S_t^d\left(1+E_k^{gT}\right)}{(1+F_t)}+\frac{\bowtie S_t^d}{(1+F_t)}\right]}{R/\forall}\right\}^{-R/\forall}$$

$R/\forall$ is Risk aversion (degree)

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[7]It is to be noted that E_b^{gT} is exactly equal to the change in the value of saved money because of changes in the price level from one to the other period Pt/Pt+1

A. Utility Maximization when Loyal Depositors are paid only their Due Share

Both kind of deposit holders are expected to have the same usefulness. The price paid to faithful depositors for a rather longer time of consumption postponement is the higher returns they earn in contrast to fickle depositors.

So $$S_f^d = S_l^d$$

Replacing equation 10 and 11 in it

$$= -(n-\text{e})\left\{\frac{\left\{E_l^{gT}\right\}^{-R/\forall}(1-T)\left[\frac{(1-\bowtie)S_t^d\left(1+E_k^{gT}\right)}{(1+F_t)}+\frac{\bowtie S_t^d}{(1+F_t)}\right]}{R/\forall}\right\}^{-R/\forall}$$

Substituting in it the returns rate paid to fickle and loyal depositors (E_f^{gT}, E_l^{gT}) respectively from equation 8 and 9, and simplifying:

$$-\frac{(n-\hbar)^{1+R/\forall}\left\{(\bowtie S_t^d)\, E_b^{gT}\right\}^{-R/\forall}}{R/\forall} = -\frac{(n-\text{e})^{1+R/\forall}\left\{\left((1-\bowtie)S_t^d\right)\left\{E_k^{gT}\right\}\right\}^{-R/\forall}}{R/\forall}$$

As the portion of total bank portfolio invested in capital market is $(1-\bowtie)S_t^d$, therefore, we replace it with K_d^I (notation for capital produced) and the remaining amount of saving ($\bowtie S_t^d$) as $1 - K_d^I$.

Solving

$$\frac{(n-\hbar)^{1+R/\forall}\left\{(1-K_d^I)\, E_b^{gT}\right\}^{-R/\forall}}{R/\forall} = \frac{(n-\text{e})^{1+R/\forall}\left\{(K_d^I)\left\{E_k^{gT}\right\}\right\}^{-R/\forall}}{R/\forall}$$

As we know the E_k^{gT} is the capital return rate, we put its value from equation 5, where its degree is determined in the milieu of total investment (both public and private together) in capital market

Rearranging, we get

$$K_d^{I\,-R/\forall} = \frac{(n-\hbar)^{1+\frac{R}{\forall}}\,(E_b^{gT})^{-R/\forall}}{(n-\text{e})^{1+R/\forall}\{P(N_{\sigma c})\}^{R/\forall}}\left\{\frac{1}{1+\left[\frac{(n-\hbar)^{1+\frac{R}{\forall}}\,(E_b^{gT})^{-R/\forall}}{(n-\text{e})^{1+R/\forall}\{P(N_{\sigma c})\}^{R/\forall}}\right]}\right\}$$

As $\frac{R}{\forall}$ is risk outline of house holders, it should be figured in better notation of single staging instead of its divisible characteristic. Therefore, we replace this term with a single notation of ß in the above equation, besides replacing K_d^I with $(1-\bowtie)S_t^d$ as the total ventures taken in the capital market.

$$(12) \qquad (1-\bowtie)S_t^d = \left[\frac{n-\text{e}}{n-\hbar}\right]^{1+1/ß} E_b^{gT}\{\mathrm{P}(N_{\sigma c})\}\left\{\frac{1}{1+\left[\left[\frac{n-\text{e}}{n-\hbar}\right]^{1+ß} E_b^{gT}\{\mathrm{P}(N_{\sigma c})\}\right]}\right\}$$

B. Utility Maximization for Depositors when Loyal Depositors Are Paid More than their Due Share as Reward for their Loyalty

In the second phase, we assume that the time and demand deposits are not managed separately. Instead, we take into consideration the total bank's portfolio (all deposits when $S_t^{GB} = 0$) without a pre-tag of demand or time deposits.

We add an extra share equal to $\frac{\bowtie S_t^d}{(1+F_t)}$ E_b^{gT} in the returns of loyal depositors the bank offers them. In such scenario, equation 8 for fickle depositors will remain the same; while equation 9 for loyal depositors will take the following shape (equation 9a):

$$(9a) \qquad E_l^{gT*} = \left\{\frac{(1+F_t)A-(\bowtie)S_t^d}{(1+F_t)} E_k^{gT} + \frac{\bowtie S_t^d}{(1+F_t)} E_b^{gT}\right\}\left\{\frac{1}{(n-\text{e})}\right\}$$

As discussed above, the utilities of those who prefer liquidation of the time deposits on maturity and those who withdraw money before the maturity time have the same dynamics; we put the utility functions of both the representative deposit holders as identity equation.

$$S_f^d = S_l^d$$

Repeating the same process of utility maximization with only one modification as

$$= -(n-\text{e})\left\{\frac{\{E_l^{gT}\}(1-T)\left[\frac{(1-\bowtie)S_t^d}{(1+F_t)} + \frac{\bowtie S_t^d\left(1+E_b^{gT}\right)}{(1+F_t)}\right]}{R/\forall}\right\}^{-R/\forall}$$

Substituting in it the returns rate paid to fickle and loyal depositors (E_f^{gT}, E_l^{gT}) respectively from equation 8 and 9a, and simplifying:

$$= -(n-\text{e})\left\{\left\{\frac{(1+F_t)A-\bowtie S_t^d}{(1+F_t)} E_k^{gT} + \frac{\bowtie S_t^d}{(1+F_t)} E_b^{gT}\right\}\left\{\frac{1}{(n-\text{e})}\right\}(1-T)\left[\frac{(1-\bowtie)S_t^d}{(1+F_t)} + \frac{\bowtie S_t^d\left(1+E_b^{gT}\right)}{(1+F_t)}\right]\right\}^{-\frac{R}{\forall}}$$

It is clear from the above equation that loyal depositor is bestowed not only with the returns from capital market $\frac{(1+F_t)A-(\bowtie)S_t^d}{(1+F_t)} E_k^{gT}$ but also rewarded with portion $\frac{\bowtie S_t^d}{(1+F_t)}$ E_b^{gT} from money market's earning.

Rearranging, cancelling out the inflation denominator from both sides of equation and simplifying exactly as above:

$$\frac{(\mathrm{n}-\hbar)\left\{\frac{\bowtie \mathrm{S}_{\mathrm{t}}^{\mathrm{d}}}{1}\,\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}\left\{\frac{1}{(\mathrm{n}-\hbar)}\right\}\right\}^{-\mathrm{R}/\forall}}{\mathrm{R}/\forall}=-\frac{(\mathrm{n}-\mathrm{e})\left\{\left\{\frac{(1-\bowtie)\mathrm{S}_{\mathrm{t}}^{\mathrm{d}}}{(1)}\,\mathrm{E}_{\mathrm{k}}^{\mathrm{gT}}+\frac{\bowtie \mathrm{S}_{\mathrm{t}}^{\mathrm{d}}}{1}\,\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}\right\}\left\{\frac{1}{(\mathrm{n}-\mathrm{e})}\right\}\right\}^{-\mathrm{R}/\forall}}{\mathrm{R}/\forall}$$

Replacing and solving for total amount of investment in capital market K_d^I and rearranging we get

(13) $$(1-\bowtie)\mathrm{S}_{\mathrm{t}}^{\mathrm{d}}=\left[\frac{\mathrm{n}-\mathrm{e}}{\mathrm{n}-\hbar}\right]^{1+1/ß}\frac{\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}}{\left\{\mathrm{P}(\mathrm{N}_{\eth c})-\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}\right\}}\left\{\frac{1}{1+\left[\frac{\mathrm{n}-\mathrm{e}}{\mathrm{n}-\hbar}\right]^{1+1/ß}\frac{\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}}{\left\{\mathrm{P}(\mathrm{N}_{\eth c})-\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}\right\}}}\right\}$$

Government Constraint of Budget

We mostly consider the budget likelihood (revenues and expenditure), seigniorages financing and government issued bonds, so forming the budget restriction of the government. One believes that whole expenditure is supplied and exogenially fixed.

As we know, the limited inter-temporal budget constraint is only the government future flows and inflows estimate, which ensure the repayment of public money borrowed. Stated otherwise, the path of the constraint mostly reflects the budget inflows, so it corresponds to the present value account.

i. Simple Budget Equation

To avoid the difference of value on the basis of changes in price level, we take debt and money balances in real terms.

(14) $$\frac{B_{D,t-1}}{P_{t-1}}\left(r_{f_{t-1}}\right)+G_t=T_t(Y_t)+\Delta\frac{B_{D,t}}{P_t}+\Delta\frac{B_{M,t}}{P_t}$$

$B_{D,t}$ is nominal debt at time period t, $B_{M,t}$ is nominal money balances at time period t, $B_{D,t-1}$ is nominal debt balance in previous time period, r_f is the risk free interest rate, , $Y_t$is aggregate income at time period t, G_t is total spending in time period t.

Replacing

$$\Delta\frac{B_{D,t}}{P_t}=\left[\frac{B_{D,t}}{P_t}-\frac{B_{D,t-1}}{P_{t-1}}\left(\frac{P_{t-1}}{P_t}\right)\right]$$

And substituting in reverse order recursively, we the following equation[8]:

(15) $$G_{t+1}-T_t(Y_t)+=\left[\frac{B_{D,t+1}}{P_{t+1}}-\frac{B_{D,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right]+\left[\frac{B_{M,t+1}}{P_{t+1}}-\frac{B_{M,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right]-\frac{B_{D,t}}{P_t}\left(r_{f_t}\right)$$

We suppose that regardless of the length of the time span stretched across, the present debt value cannot attain a non-negative number. Stated differently, the rule is that debt rolling over to the following time

[8]Based on the above gesticulation, a more generalized budget constraint can be developed for more enlarged time periods.

period follows a condition whereby the ponzi scheme is standard. Under such circumstances, the government would rather be in a perpetual state of debt forwarding the debt to the next temporal period. This picture helps one to observe a quite fascinating aspect of Pakistan's economic situation. Pakistan's government taxes three sectors of the economy solely; the fourth (agricultural) sector is tax free. Following Chene (2006) and adjusting for our model by presuming that agricultural output always represents a fixed share of total output, we split the total taxable income in two halves:

1. The agriculture sector which is exempted from total tax Y_t^a and
2. The rest of the economy being taxed $\tau\ (Y_t - Y_t^a)$

We consider $T_t(Y_t) = \tau(Y_t - Y_t^a)$ being the total government revenues when agriculture sector is exempted from tax. In such scenario, the above originated government budget constrained (in equation 15) be devised as follows:

(16) $$G_{t+1} - \tau(Y_t - Y_t^a) = \left[\frac{B_{D,t+1}}{P_{t+1}} - \frac{B_{D,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right] + \left[\frac{B_{M,t+1}}{P_{t+1}} - \frac{B_{M,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right] - \frac{B_{D,t}}{P_t}\left(r_{f_t}\right)$$

Let $$Y_t^a = e\,(Y_t)$$

Then $$T_{t+1}(Y_{t+1}) = \tau(Y_{t+1} - Y_{t+1}^a) = \left[\tau\,(1-e)Y_{t+1}\right]$$

The agriculture sector accounts for a certain portion of total output in a country. The absence of tax imposition on agriculture sector reduce the expression $\tau(Y_{t+1} - Y_{t+1}^a)$ to $\tau(Y_{t+1} - (e)Y_{t+1})$ making the total output equal to $\left[(e)Y_t + (1-e)Y_t\right]$ and equation 16 as below:

(16a) $$G_{t+1} - \tau(Y_{t+1} - eY_{t+1}) = \left[\frac{B_{D,t+1}}{P_{t+1}} - \frac{B_{D,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right] + \left[\frac{B_{M,t+1}}{P_{t+1}} - \frac{B_{M,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right] - \frac{B_{D,t}}{P_t}\left(r_{f_t}\right)$$

We need to consider the sustainability of the debt level before determining a clear link between inflation and deficit budget dynamics. To talk about, the idea of debt sustainability looks realistic as it tells us that, apart from keeping a balance budget, the government may honor the promises it made over a period of time. But the actual world, in which uncertainty is essentially the norm rather than unusual, shows quite a different picture from the one discussed above. The inclusion of uncertainty would help the model to be more reasonable. Still, it is usual to highlight the dynamics of public deficit finance before heading toward that objective. Those who buy risk free government assets to help to cover the budget deficit are essentially giving government their purchasing power instead of their own.

Presumably, this purchasing power is a moral obligation on the side of the government to value at least the same if not more when delivered back to the households. To keep the purchasing power of all current time lenders, the government thus pays a positive interest rate in future to them. Still, greater focus should be on the uncertainty of future and debt sustainability in order to solve them. From the tranversality viewpoint, the interest rate as compensation has its incidence, as in the preceding equations; but, market interest rate has insufficient application for future uncertainty and sustainability. Therefore, we propose the debt to money balances ratio (for replacing debt figures with money given) to avoid the debt related dynamics of the model, especially the debt sustainability, so keeping the model simple and avoiding unnecessary complexity. Thus, deeming the debt to money ratio as constant and denoting it by Џ, we can get a value for debt balances in terms of money balances through simplification as below:

$$\text{Џ} = \frac{B_{D,t}}{B_{M.t}}$$

(16b) $$B_{D,t} = \text{Џ}\, B_{M,t}$$

Accommodating the debt balances in terms of money balances, the government budget constraint (equation 16) will take the following shape:

(17) $$G_{t+1} - \tau\{Y_{t+1} - (e)Y_{t+1}\} = \left[\text{Џ}\frac{B_{M,t+1}}{P_{t+1}}\right] + \left[\frac{B_{M,t+1}}{P_{t+1}} - \frac{B_{M,t}}{P_t}\left(\frac{P_t}{P_{t+1}}\right)\right]$$

As the economic literature argues, the budget expenditure (G_t) is linked with total output of the economy(Y_t). The total public expenditure is either less than or equal to it. Under some abnormal circumstances the public expenditure can also surpass the output level.

So lets $$G_t = \text{Å}\, Y_t$$

Where 'Å' can be less than one, greater than one or equal to one, signifying a level when expenditure is less than output, greater than output and equal to output, respectively. Also, expressing the seigniorages revenue[9] $\Delta B_{M,t}$ in real terms, the equation of budget constraint will take the shape as follows:

(18) $$Y_{t+1}(\text{Å}) - Y_{t+1}(\tau - e) = \left[\text{Џ}B_{m,t+1} + B_{m,t+1} - B_{m,t}\left(\frac{P_t}{P_{t+1}}\right)\right]$$

$*\frac{\Delta B_{M,t}}{P_t} = \Delta B_{m,t}$

We consider that money balances follow a steady state trend and grow at fixed rate. Then the fixed increase in money balances can be coated as below:

$$B_{m,t+1} - \text{ȝ}B_{m,t} = B_{m,t}$$

$$B_{m,t+1} = B_{m,t}(1 + \text{ȝ})$$

$$B_{m,t+1} = \Gamma\, B_{m,t}$$

The steady state economy assumption holds the grounds for output growth in the same fashion as for the money growth.

$$Y_{g_{,t+1}} - \text{ȣ}Y_{g_{,t}} = Y_{g_t}$$

Let $$(1 + \text{ȣ}) = \text{ф}$$

$$Y_{gt+1} = \text{ф}Y_{g_t}$$

Encompassing the fixed output and money growth considerations in budget constraint equation and replacing $E_b^{gT} = \frac{P_t}{P_{t+1}}$

[9] See Appendix III for seigniorage Notes

$$Y_{t+1}(\text{Å} - \tau + e) = \left[\text{ Џ}\Gamma B_{m,t} + \Gamma B_{m,t} - B_{m,t}\left(\frac{P_t}{P_{t+1}}\right)\right]$$

$$Y_{g_t}(\text{Å} - \tau + e) = B_{m,t}\left[\Gamma(1 + \text{Џ}) - \mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}\right]/\phi$$

Assuming the money growth exactly equal to output growth, we replace the mathematical expression of change in money supply (Γ) for change in aggregate perdition (ϕ).

$$(19) \qquad Y_{g_t}(\text{Å} - \tau + e) = B_{m,t}\left[(1 + \text{Џ}) - \frac{\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}}{\Gamma}\right]$$

Given that the level of expenditure and taxes are externally determined and $\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}$ is a positive or negative price shock, the above equation shows that the output depends on money balances $B_{m,t}$ and growth rate Γ. In the following segment we will determine the upshot for money balances and growth rate.

ii. Contriving 'Money Balances' and 'Growth Rate' for inserting in Budget constraint Equation

As the money balances $B_{m,t}$ and growth rate Γ are endogenous in nature and its values are determined within the economic system, therefore in this segment we will set up the money balances and growth rate equations. Once the expressions for (a) money balances and (b) growth rate are mathematically determined, we will place it back in the above budget equation (17) for final treatment of our analysis.

A. Culmination of money Balances

To define the money balances in term of money demanded to finance transaction, we consider the current account cash into consideration. As we know from equation (6) that total stock of bank portfolio is divided into two mutually exclusive scraps, the one held by banks in the current account $\bowtie S_t^d$ and the other invested in the capital market $(1-\bowtie)S_t^d$.

Money balance = total earning after tax * cash in hand[10]

$$(20) \qquad \mathrm{B}_{\mathrm{m,t}} = (1 - \mathrm{T})(1 - \mathrm{N}_{ðc})(\text{Ƥ})\mathrm{p}_{\mathrm{KT}}\left[\bowtie \mathrm{S}_{\mathrm{t}}^{\mathrm{d}}\right]$$

Where T is taxes, $(1 - N_{\sigma c})(\text{Ƥ})p_{KT}$ as previously defined in equation (4) is equal to total earning of the economy as all the householders are bestowed with one source of income (wages). While, the portion of total saving kept in current account for day to day transactions or the deposits liquidated before the time of maturity for consumption purposes is equal to $(\bowtie S_t^d)$.

As total earnings are equal to total savings and total savings are equal to total deposits, therefore $\mathrm{S}_{\mathrm{t}}^{\mathrm{d}}$ is equal to 1. Substituting in equation (6), the current account balances $(\bowtie \mathrm{S}_{\mathrm{t}}^{\mathrm{d}})$ are equal to the following expression

$$(21) \qquad \bowtie \mathrm{S}_{\mathrm{t}}^{\mathrm{d}} = 1 - (1 - \bowtie \mathrm{S}_{\mathrm{t}}^{\mathrm{d}})$$

Taking the notation of $(1 - \bowtie \mathrm{S}_{\mathrm{t}}^{\mathrm{d}})$ from the previously defined equation (13) of section II as follows:

[10] depositors current account balances are included while their investment in bonds and capital markets is excluded

$$(1-\bowtie)S_t^d = \left[\frac{n-e}{n-\hbar}\right]^{1+1/\beta} \frac{E_b^{gT}}{\{\mathrm{P}(N_{\sigma c}) - E_b^{gT}\}} \left\{ \frac{1}{1+\left[\frac{n-e}{n-\hbar}\right]^{1+1/\beta} \frac{E_b^{gT}}{\{\mathrm{P}(N_{\sigma c}) - E_b^{gT}\}}} \right\}$$

Reducing the expression $\left[\left(\frac{n-e}{n-\hbar}\right)^{1+1/\beta} \frac{E_b^{gT}}{\mathrm{P}N_{\sigma c}-E_b^{gT}}\right]$ to be equal to $'\Xi'$ as follow:

(21a) $$\Xi = \left[\left(\frac{n-e}{n-\hbar}\right)^{1+1/\beta} \frac{E_b^{gT}}{\mathrm{P}N_{\sigma c}-E_b^{gT}}\right]$$

And then replacing this equation (21a) in equation 21, we get the following expression for current balances:

(22) $$\bowtie S_t^d = 1-(1-\bowtie S_t^d) = 1-(\Xi \frac{1}{1+\Xi}) = \left(\frac{1+\Xi-\Xi}{1+\Xi}\right) = \frac{1}{1+\Xi}$$

Replacing equation (22) in the money balances equation (20), we get an expression as follows:

(23) $$B_{m,t} = \left[(1-T)(1-N_{\eth c})\,(\mathrm{P})p_{KT}\right]\frac{1}{1+\Xi}$$

B. Evolution in Capital Stock (Output Growth Rate)

As the constant growth rate (Γ) in this steady state model is based on Cobb Douglas production function, therefore, following Solow growth model (1956) with constant per capita growth we define our profit maximization function for representative firm as follows:

$$\pi_{max\,(p_K,h_l)} = \mathrm{P}\left(\bar{g}_k^{1-N_{\sigma c}}\, p_{kt}^{N_{\sigma c}}\, h_{lt}^{1-N_{\sigma c}}\right) - R_w\left(h_{lt}^{N_{\sigma c}}\right) + k_r(p_{Kt})$$

During one-time period, the capital stock, the population and consequently the size of available work force remains the same. As the capital generation in the next time period depends on the portion of income saved by the households in current time period and invested in capital markets by banks.

Out of the Income invested in capital markets in time period t, the undertaken successful projects convert one unit of labor in the start of a time period t to one unit of capital by the end of that time period. In the Solow model, the stock of capital in time period t+1 depends on the amount of capital in the previous time period besides the amount of capital generated by the end of previous time period.

As we know that in our model, the portion of total bank portfolio invested in capital market is equal to $(1-\bowtie)S_t^d$, therefore, the capital scenario in time period t exhibits the following illustration for capital accumulated in time period t+1

(24) $$p_{K_{t+1}} - p_{K_t}(1-\Upsilon) = (1-\bowtie)S_t^d\left\{\mathrm{P}\left(\bar{g}_k^{1-N_{\sigma c}}\, p_{kt}^{N_{\sigma c}}\, h_{lt}^{1-N_{\sigma c}}\right)\right\}$$

Capital depreciates fully in one-time period and there is no lagged dependency. Therefore, the part of capital (Υ) depreciated in production process during the start and end time period t will become equal to one, and the portion $p_{Kt}(1-\Upsilon)$ becomes equal to zero. It is worth noting that the value of Υ reaches to 1 by the end of the year and therefore the time period (t) capital stock completely vanish away by the end of ‘t’ time period. However, the capital complete deterioration takes a span of one year, it doesn’t finish at the start of time period ‘t’.

Taking Υ equal to one, making the insured income stationary and defining the capital in the terms of per capita, the above equation will reduce to the following form:

(25) $$p_{K_{t+1}} = (1-\bowtie)S_t^d \left\{\text{Ƥ}\left(\bar{g}_k^{1-\mathrm{N}_{\sigma c}}\, p_{kt}^{\mathrm{N}_{\sigma c}}\, h_{lt}^{1-\mathrm{N}_{\sigma c}}\right)\right\}$$

We know from equation (4) that the total production in time period t based on the available resources (total capital) p_{KT} and given labour productivity Ƥ after the tax deduction is equal to:

$$(1-t)\{1-\mathrm{N}_{\sigma c}\}\,(\text{Ƥ})p_{KT}$$

Therefore

(26) $$\text{Ƥ}\left(\bar{g}_k^{1-\mathrm{N}_{\sigma c}}\, p_{kt}^{\mathrm{N}_{\sigma c}}\, h_{lt}^{1-\mathrm{N}_{\sigma c}}\right) = (1-t)\{1-\mathrm{N}_{\sigma c}\}\,(\text{Ƥ})p_{KT}$$

Replacing equation (26) in the above equation (25) we get the following expression for next time period capital:

(27) $$p_{K_{t+1}} = (1-\bowtie)S_t^d\,(1-t)\{1-\mathrm{N}_{\sigma c}\}\,(\text{Ƥ})p_{KT}$$

By replacing $(1-\bowtie)S_t^d = \Xi\frac{1}{1+\Xi}$ from previously defined equation (22) in the above equation (27), we get:

(27) $$p_{K_{t+1}} = \Xi\frac{1}{1+\Xi}\,(1-t)\{1-\mathrm{N}_{\sigma c}\}\,(\text{Ƥ})p_{KT}$$

We can acquire the gross rate of equilibrium 'capital growth' through dividing the future capital by the current period capital ($\Gamma = \frac{p_{Kt+1}}{p_{Kt}}$). Replacing the expression for p_{Kt+1} from equation (27), we can find the balance path of equilibrium growth rate as below:

$$\frac{p_{K_{t+1}}}{p_{K_t}} = (\Xi\frac{1}{1+\Xi})(1-t)\{1-\mathrm{N}_{\sigma c}\}\,(\text{Ƥ})p_{KT}/p_{KT} = \Gamma$$

(28) $$\Xi = \frac{\Xi(1-t)\{1-\mathrm{N}_{\sigma c}\}\,(\text{Ƥ})}{1+\Xi}$$

Now finally getting back to the government budget constraint equation (19) and replacing the money balances ($\mathrm{B}_{m,t}$) and change in capital stock (growth rate Γ) expressions in it from equation (23) and equation (28) respectively:

$$(\text{Å}-\tau+e) = \left\{\frac{1}{1+\Xi}(1-t)(1-\mathrm{N}_{\sigma c})\right\}\left[(1+\text{ Џ}) - \frac{\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}}{\frac{\Xi(1-t)(1-\mathrm{N}_{\sigma c})(\text{Ƥ})}{1+\Xi}}\right]$$

Simplifying

(29) $$(\text{Å}-\tau(1-e)) = \left\{\frac{1}{1+\Xi}(1-t)(1-\mathrm{N}_{\sigma c})\right\}(1+\text{Џ}) - \left(\frac{\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}}{\Xi\,(\text{Ƥ})}\right)$$

In addition to the revenues $\tau\,\{1-e\}$ generated through imposing taxes on the total earning of all the inhabitants, the right hand side of budget equation (29) shows the added tax load needed to finance the government budget when there is a gap between total revenues and total expenditure. This portion of revenues is the segment which is to be finances by means other than direct taxation, i.e. seigniorages.

Analysis for corridors of Balanced Output Growth and Inflation

1. *Growth Dynamics*

As in all our paper, the decision making process of fiscal and monetary policy authorities are based on many exclusive and exogenously determined assumptions. Therefore, a coordinated decision making on the part of both monetary and fiscal authorities will be a land mark achievement and a perfectly constructive outcome for the economy. The no coordination policy can end up as in high inflation and low growth outcomes. As happened in Pakistan, all efforts on the part of the government to augment growth ended up in high inflation tendencies during the first decade of 21st century.

In our study, the focus is mainly on the depositors' risk taking behavior. The reason behind choosing this mode is that saving and investment decisions on part of householders are affected by both monetary and fiscal policies. The saving decision of householders is more affected by fiscal policy considerations, while interest rate dynamics are mainly associated to monetary policy upshots, linking the two policies in an operative manner. In our case, the connection between two policies exhibit more prominent linkage as all householders are savers and all savers are deposit holders.

To start, we first take into account the households' risk taking behavior to spot the different cases of 'uniquely determined growth equilibrium' and 'multiple growth equilibrium'. All of this is established on the basis of risk aversion themes of the households.

A. uniqueness of the equilibrium growth

Now, to see the uniqueness of the equilibrium growth level, we assume that such possibility exists only if the agents exhibit a high degree of risk aversion. We take into account the government budget constraint (equation 29) and solve it for two different situations: One, when the returns on deposits ($\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}$) is equal to zero, and second, when the return on deposits is equal to capital rent $Ꝑ(N_{\sigma c})$.

$$(\text{Å} - \tau(1-e)) = \left\{\frac{1}{1+\Xi}(1-t)(1-\mathrm{N}_{\sigma c})\right\}(1+\text{Ꞁ}) - \left(\frac{\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}}{\Xi\,(Ꝑ)}\right)$$

Where Ξ is defined in equation (21a) as: $\Xi = \left[\left(\frac{\mathrm{n}-\mathrm{e}}{\mathrm{n}-\hbar}\right)^{1+1/ß} \frac{\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}}{ꝐN_{\sigma c}-\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}}\right]$

i. Solving government budget constraint (equation 29) for $\mathbf{E}_{\mathbf{b}}^{\mathbf{gT}} = \mathbf{0}$

(A1) $$\text{Å} = \mathrm{T}\,(1-e) + (1-t)(1-\mathrm{N}_{\sigma c})(\text{Ꞁ}+1)$$

ii. Solving government budget constraint (equation 29) for $\mathbf{E}_{\mathbf{b}}^{\mathbf{gT}} = Ꝑ(N_{\boldsymbol{\sigma c}})$

(A2) $$\text{Å} + \left(\frac{\mathrm{N}_{ðc}}{\Xi}\right) = \mathrm{T}\,(1-\mathrm{e}) + [(1-\mathrm{t})(1-\mathrm{N}_{ðc})(\text{Ꞁ}+1)]$$

It is obvious from equation A3 & A4 that when the return rate to depositors is equal to the capital rate ($i.e,\ \mathrm{E}_{\mathrm{b}}^{\mathrm{gT}} = ꝐN_{\sigma c}$), then the total budget revenues and seigniorages generated are less than expenditure. On the other hand, a zero return's rate on deposits equal to zero ($i.e,\ \mathrm{E}_{\mathrm{b}}^{\mathrm{gT}} = 0$), will make the total government budget expenditure (Å) equal to the revenues toll. The reason behind is the previously discussed value of deposit rate ($\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}$), which is equal to the inverse of inflation(P_t/P_{t+1}). Thus, making the deposit rate (DR) equal to zero indicates no inflation and the public revenues are just equal to government's expenditure, while making (DR) equal to capital lending rate means deficit budget gap is more than the seigniorages can fill and needs an additional $\frac{N_{\sigma c}}{\Xi}$ for balancing budget. It can be expressed by combining equation A1 and A2 as follows:

$$\tau(1-e)+(1-t)(1-\mathrm{N}_{\sigma c})(\text{Ḭ}+1) > \text{Å} > \tau(1-e)+\left[(1-t)(1-\mathrm{N}_{\sigma c})(\text{Ḭ}+1)-\left(\frac{\mathrm{N}_{\sigma c}}{\Xi}\right)\right]$$

Taking into account the above two situations when $\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}} = (0)$ and $\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}} = (\text{Ƥ}N_{\sigma c})$, we check for the outcomes of unique and multiple growth equilibrium on the basis of representative deposit holder's risk taking preferences.

The first situation is when deposit rate is equal to zero ($\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}} = 0$) and the agents exhibit a truly risk aversive behavior (not letting bank to invest in risky money market ventures), setting ẞ greater than zero.

As we know that the budget constraint equation (equation 29) has two parts, namely the 'primary budget balance' and the 'seigniorages'. As proved in the above two equations (A1 and A2), the budget is balanced when deposit rate is equal to zero ($\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}} = 0$) and depositor are just paid with the inverse of inflation P_t/P_{t+1}. While negative when the deposit rate is equal to capital returns.

The first difference of equation A1's right hand side is lower than zero and the left hand side is greater than zero. There exists equilibrium growth when the deposit rate is less than the capital rent rate provided that that risk aversion is greater than or equal to zero.

To see whether the equilibrium is 'exclusive', or there exist 'multiple growth equilibriums', we take into account the level of depositor's risk aversion and first difference of primary budget. There will be a unique equilibrium as long as the risk aversion of agents is greater than zero, less than capital rate and the first different of primary budget balance is greater than the first different of seigniorage revenues.

In our case, it can be readily verified from equation (21a) of part II that $\partial\frac{\Xi}{\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}} > 0$ while $\partial\frac{\Xi}{\mathrm{n}-\hbar} < 0$ by taking the derivation of Ξ with respect to $\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}$ and then with respect to the fickle depositors $\mathrm{n}-\hbar$

Taking First derivative of equation 21a with respect to $\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}$:

$$\frac{\text{Ƥ}N_{\sigma c}\left(\frac{\mathrm{n}-\text{Ɵ}}{\mathrm{n}-\hbar}\right)^{1+\frac{1}{\beta}}}{\left(\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}-\text{Ƥ}N_{\sigma c}\right)^{2}}$$

Taking First derivative of equation 21a with respect to $\mathrm{n}-\hbar$:

$$-\frac{\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}(\mathrm{n}-\text{Ɵ})\left(\frac{\mathrm{n}-\text{Ɵ}}{\mathrm{n}-\hbar}\right)^{\frac{1}{\beta}}\left(1+\frac{1}{\beta}\right)}{(\mathrm{n}-\hbar)^{2}\left(-\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}+\text{Ƥ}N_{\sigma c}\right)}$$

B. Growth Response to 'Budget Deficit' and to Resultant 'Inflation'

Now we want to check the kind of variation deficit financing of budget can bring in growth prospects of the economy. In this context, first we will directly test changes in capital growth (presumably equal to output growth) due to change in expenditure (Direct).

i. Change in capital Growth to change in expenditure (Direct)

To find direct response of changes in public expenditure in of capital growth (capital growth leads to output growth), we will express the budget equation (29) in terms of capital growth as follows:

$$\text{(A9)} \qquad \Gamma = \frac{\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}}{\left\{\frac{1}{1+\Xi}(1-t)(1-\mathrm{N}_{\sigma c})\right\}(1+\text{Ų}) - [\text{Å} - \tau(1-e)]}$$

Taking derivative with respect to change in budget expenditure:

$\partial\frac{\Gamma}{\text{Å}} > 0$[11]

The positive association between capital growth and expenditure (keeping revenue change equal to zero) may be the result of low initial inflation and the households slant of considering money as substitute to capital.

ii. Change in capital Growth in response to change in inflation (Indirect)

a. when agents are Risk Aversive (indirect)

Keeping the tax revenue unchanged, we will check change in growth due to change in seiniorages when the risk aversion of the house holders is greater than zero.

first we will take derivative of the balance growth and inflation determinant (Ξ)with respect to inverse of inflation. It is evidently clear that ($\partial\frac{\Xi}{\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}} < 0$) when ß is not negative. In second phase, we will put the value of Ξ in the equation of capital growth (which is presumable equal to balance output growth);

$$\text{(A10)} \qquad \Gamma = \frac{\left[\frac{\mathrm{n}-\text{ɐ}}{\mathrm{n}-\hbar}\right]^{1+1/\text{ß}}(1-t)\{1-\mathrm{N}_{\sigma c}\}\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}(\text{Ƥ})}{(\text{Ƥ}\mathrm{N}_{\sigma c} - \mathrm{E}_{\mathrm{b}}^{\mathrm{gT}})(1+\Xi)}$$

And check the change in balance capital growth Γ due to change in price level because a price level is positively related to seigniorage financing of budget deficit.

It reveals that if the existing level of inflation is low, decisions of expansionary policies on the part of fiscal authorities can be fruitful in channelizing the money held by the householders to investment and consequent capital accumulation, leading to higher growth outcomes.

b. when agents are not risk aversive (indirect)

Using the same procedure as above except that now the value of risk aversion ß is negative we will put the value balance growth and inflation determinant (Ξ)in the equation of capital:

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11 $\frac{\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}}{(1+\text{Ų})\frac{1}{1+\Xi}(1-\mathrm{t})(1-\mathrm{N}_{ðc})\left[1-\frac{-(1-\mathrm{e})\mathrm{T}+\text{Å}}{(1+\text{Ų})\left\{\frac{1}{1+\Xi}(1-\mathrm{t})(1-\mathrm{N}_{ðc})\right\}}\right]^2}$

(A11) $$\Gamma = \frac{\left[\frac{\mathrm{n}-\mathrm{e}}{\mathrm{n}-\hbar}\right]^{1+1/\beta}(1-t)\{1-\mathrm{N}_{\sigma c}\}\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}\,(\mathrm{P})}{(\mathrm{P}\mathrm{N}_{\sigma c}-\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}})(1+\Xi)}$$

And check the change in balance capital growth Γ due to change in price. This time, in line with most of the economic studies on the subject, the result indicates a higher inflation guides to lower than otherwise expected growth and capital accumulation on the face of higher demand for real balances holdings. This can be readily verified from our result of $(\partial\frac{\Gamma}{\mathrm{E}_{\mathrm{b}}^{\mathrm{gT}}} > 0)$ for this particular upshot when agents are more risk takers.

CONCLUSION

In emerging countries, too ambitious claims on government resources could result in structural imbalances like limited capital creation and crowding out consequences. Observed in Pakistan during the past three decades, ongoing fiscal deficits have degraded real sector performance and hindered sustainable production growth. Driven by disproportionately high non-developmental expenditure, excessive debt servicing, and inadequate tax income, fiscal mismanagement continues even with several policy initiatives.

Emphasizing the importance of financial intermediaries as main drivers of capital allocation, the study tries to grasp how monetary and fiscal policies affect output growth. The study emphasizes the need of bank lending decisions, which depend on the risk-taking activities of depositors. Because depositors are ready to tolerate more risk, banks can direct funds into more profitable, if riskier, businesses, hence promoting increased output growth.

Our results show a negative link between taxes and growth since higher taxes diminish private consumption and savings, therefore affecting the resource efficiency in emerging countries. When taxes increase without commensurate adjustments in public expenditure, households curtail consumption and investors scale their savings, therefore limiting capital creation. When deposit rates are less than capital returns, growth slows down; still, equilibrium is kept as long as depositors show reasonable risk aversion.

The study also reveals that changes in budget expenditure favorably affect development since public spending can boost private sector activity when households see money as a replacement for capital. In these situations, more public spending moves cash reserves into investments, therefore improving capital building and promoting long-term development.

The research basically comes to the conclusion that balancing public and private sector dynamics depends on efficient policy cooperation. Policymakers may build a sustainable road for increased output and capital accumulation in emerging nations like Pakistan by encouraging an environment whereby financial intermediaries may effectively allocate resources and match fiscal policies with long-term economic targets.

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